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Semantic Information and the Trivialization of Logic: Floridi on the Scandal of Deduction

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Abstract: In this paper we discuss Floridi’s views concerning semantic information in the light of a recent contribution (in collaboration with the present author) [1] that defies the traditional view of deductive reasoning as “analytic” or “tautological” and construes it as an informative, albeit non-empirical, activity. We argue that this conception paves the way for a more realistic notion of semantic information where the “ideal agents” that are assumed by the standard view can be indefinitely approximated by real ones equipped with growing computational resources.

Keywords: semantic information; Cohen–Nagel paradox; tractability; analytic-synthetic distinction

1. Introduction

“Philosophical work on the concept of [...] information is still at that lamentable stage when disagreement affects even the way in which the problems themselves are provisionally phrased and framed” [2]. Thus Luciano Floridi, in his entry on “Semantic conceptions of information” in the Stanford Encyclopedia of Philosophy, takes a snapshot of the state of the art in the philosophy of information A.D. 2005. As early as 1953, a few years after the appearance of his “Mathematical Theory of Communication”, Claude Shannon had warned that the new area of Information Theory was surrounded by some conceptual and terminological confusion that prompted for further clarification and differentiation:

The word “information” has been given many different meanings by various writers in the field of information theory. It is likely that at least a number of these will prove sufficiently useful in certain applications to deserve further study and permanent recognition. It is hardly
to be expected that a single concept of information would satisfactorily account for the numerous possible applications of this general field [3].

Half a century later, Floridi’s main contribution has probably been that of taking up Shannon’s challenge from a philosophical perspective, by engaging in the bold project of cleaning up the area and taming the “notoriously polymorphic and polysemantic” [2], although increasingly pervasive, notion of information. This ambitious program has been carried out to a considerable extent in a series of papers and, above all, in his main work on this subject, *The Philosophy of Information* [4].

In this paper we discuss Floridi’s views concerning semantic information in the light of a recent contribution (in collaboration with the present author) [1] that defies the traditional view of deductive reasoning as “analytic” or “tautological” and construes it as an informative, albeit non-empirical, activity. We argue that this conception paves the way for a more realistic notion of semantic information where the “ideal agents” that need to be assumed to defend the standard view can be indefinitely approximated by real ones equipped with growing computational resources. We also argue that this conception offers a partial vindication of the Kantian notion of “synthetic a priori” even in the allegedly trivial domain of propositional logic. (A similar vindication of Kantian ideas, but restricted to the notoriously harder domain of quantification logic, is the leitmotiv of Hintikka’s well-known book *Logic, Language Games and Information* [5].)

We start, in Section 2, by presenting the “received view” that logical inference is tautological, in the sense that it does not increase information. In this view, the conclusion can be seen to be true, given the truth of the premises, by the very meaning of the logical operators and the corresponding judgment that the conclusion follows from the premises is therefore “analytic”, a purely linguistic truth that we learn by learning the language. Accordingly, the “semantic information” carried by the conclusion—that is, the information that it carries by virtue of the meaning of the logical operators—must be contained in the information carried by the premises. In Section 3, we discuss various anomalies of the received view—the Bar-Hillel–Carnap paradox, the “scandal of deduction”, the problem of logical omniscience—and, in Section 4, Floridi’s approach to them as it emerges from [6–8]. In Section 5 we present our new approach based on an informational semantics for the logical operators. This consists in fixing their meaning in terms of the information that is actually possessed by an agent. The main idea is that their classical meaning, being based on information-transcending notions, such as classical truth and falsity, is not apt to justify the claim that inferences that are “analytic”—*i.e.*, licensed by this very meaning—are also “tautological”, *i.e.*, informationally trivial. By contrast, the central semantic notions of informational semantics are those of “actually possessing the information” that a given sentence is true, respectively false, and the meaning of the logical operators is defined exclusively in terms of these notions. As a consequence, inferences that are “analytic” according to the informational meaning of the operators turn out to be also “tautological” in a particularly strict sense—that an agent that actually possesses the information carried by the premises actually possesses also the information carried by the conclusion—and this is confirmed by the fact that the corresponding consequence relation is computationally feasible. Non-analytic (or “synthetic”) inferences, on the other hand, are characterized by the fact that they essentially require the use of “virtual information”, in the form of provisional hypothetical information that is not actually possessed by the agent who makes the inference, such as the one that plays a crucial role in the “discharge rules” of Gentzen-style Natural
Deduction (see [9] for an excellent introduction). Gradually allowing for the nested use of such virtual information naturally leads to define a hierarchy of increasingly informative inferential systems that indefinitely approximate classical propositional logic.

2. The Received View

According to the received view, logical deduction never increases (semantic) information. This tenet clashes with the intuitive idea that deductive arguments are useful just because, by their means, we obtain information that we did not possess before. However, it allowed philosophers and mathematicians to justify their view of logic and mathematics as infallible activities that are not subject to the tribunal of experience.

2.1. Logical Empiricism and the Trivialization of Logic

One of the trademarks of modern, or “logical”, empiricism was the rejection of the possibility of “synthetic a priori” judgements. (We briefly recall some philosophical terminology. A judgment is analytic when it does not extend our knowledge, but asserts something that is trivially true solely by the meaning of the words, e.g., “all bachelors are unmarried”. A synthetic judgment is one that does extend our knowledge and cannot be established by mere semantic analysis, such as “all bachelors eat in front of tv”. A judgment is a priori when its truth does not depend on experience, and a posteriori when it does. Analytic judgments are always a priori. Synthetic judgments are normally a posteriori, but Kant argued that some mathematical judgments are synthetic a priori: they do extend our knowledge but do not depend on experience.) This position was clearly stated in the manifesto of the Vienna Circle as “the basic thesis of modern empiricism”:

In such a way logical analysis overcomes not only metaphysics in the proper, classical sense of the word, especially scholastic metaphysics and that of the systems of German idealism, but also the hidden metaphysics of Kantian and modern apriorism. The scientific world-conception knows no unconditionally valid knowledge derived from pure reason, no “synthetic judgments a priori” of the kind that lie at the basis of Kantian epistemology and even more of all pre- and post-Kantian ontology and metaphysics. [...] It is precisely in the rejection of the possibility of synthetic knowledge a priori that the basic thesis of modern empiricism lies. The scientific world-conception knows only empirical statements about things of all kinds, and analytic statements of logic and mathematics [10] (p. 308).

According to the logical empiricists, the truths of logic and mathematics are necessary and do not depend on experience. Given their rejection of any synthetic a priori knowledge, this position could be justified only by claiming that logical and mathematical statements are “analytic”, i.e., true “by virtue of language”. More explicitly, they thought their truth can be recognized, at least in principle, by means only of the meaning of the words that occur in them. Since information cannot be increased independent of experience, such analytic statements must also be “tautological”, i.e., carry no information content. Hence:

The conception of mathematics as tautological in character, which is based on the investigations of Russell and Wittgenstein, is also held by the Vienna Circle. It is to be
noted that this conception is opposed not only to apriorism and intuitionism, but also to the older empiricism (for instance of J.S. Mill), which tried to derive mathematics and logic in an experimental-inductive manner as it were [10] (p. 311).

This view of deductive reasoning as informationally void is usually supported by resorting to elementary examples, as in this well-known passage by Hempel:

It is typical of any purely logical deduction that the conclusion to which it leads simply re-asserts (a proper or improper) part of what has already been stated in the premises. Thus, to illustrate this point by a very elementary example, from the premise “This figure is a right triangle”, we can deduce the conclusion, “This figure is a triangle”; but this conclusion clearly reiterates part of the information already contained in the premise. [...] The same situation prevails in all other cases of logical deduction; and we may, therefore, say that logical deduction—which is the one and only method of mathematical proof—is a technique of conceptual analysis: it discloses what assertions are concealed in a given set of premises, and it makes us realize to what we committed ourselves in accepting those premises; but none of the results obtained by this technique ever goes by one iota beyond the information already contained in the initial assumptions [11] (p. 9).

Despite its highly counterintuitive implications—at least in the ordinary sense of the word “information”, it is hard to accept that all the mathematician’s efforts never go “one iota” beyond the information that was already contained in the axioms of a mathematical theory—this view of deductive reasoning caught on and became part of the logical folklore. Most of its philosophical appeal probably lies in the fact that it appears to offer the strongest possible justification of deductive practice: logical deduction provides an infallible means of transmitting truth from the premises to the conclusion for the simple reason that the conclusion adds nothing to the information that was already contained in the premises. However, as Michael Dummett put it:

Once the justification of deductive inference is perceived as philosophically problematic at all, the temptation to which most philosophers succumb is to offer too strong a justification: to say, for instance, that when we recognize the premises of a valid inference as true, we have thereby already recognized the truth of the conclusion [12] (p. 195).

Indeed—as we shall argue in Section 3—this trivialization of logic is a philosophical overkill: a definitive foundation for deductive practice is obtained at the price of its informativeness. Logic lies on a bedrock of platitude.

2.2 Quine on Logical Truth

The conception of logical deduction as “analytic”, and therefore “tautological”, is a persistent dogma of (logical) empiricism that seems to be somewhat independent of Quine’s two dogmas [13] as well as from Davidson’s “third dogma” [14]. After all, Quine’s well-known arguments against the analytic-synthetic distinction spared the claim that the notion of analyticity had been sufficiently clarified in the restricted domain of logic. According to [13], statements that are analytic “by general
philosophical acclaim” fall into two classes: those that may be called logically true, such as “no unmarried man is married” and those that may be turned into logical truths by replacing synonyms with synonyms, such as “no bachelor is married”. Admittedly, Quine’s problem was that “we lack a proper characterization of this second class of analytic statements” for, in his view, “the major difficulty lies not in the first class of analytic statements, the logical truths, but rather in the second class, which depends on the notion of synonymy” ([13], pp. 22–32 of the 1961 edition). Four decades later, while his reservations over the notion of analyticity remained the “the same as ever”, Quine clarified that they concerned only “the tracing of any demarcation, even a vague and approximate one, across the domain of sentences in general” [15] (p. 270). But the impossibility of tracing a sharp demarcation does not exclude that there may be undebatable cases of analytic sentences. Indeed, “It is intelligible and often useful in discussion to point out that some disagreement is purely a matter of words rather than of fact” [15] (p. 270). The so-called “logical laws” are the most natural candidates for such paradigmatic examples of analytic sentences: it seems almost uncontroversial that a disagreement about a logical truth can always be reduced to a disagreement about the meaning of some logical word that occurs in it.

In fact, in The Roots of Reference Quine had already suggested that, in order to fit the undisputed cases of analytic sentences, one may provide a rough theoretical definition of analyticity by saying that (i) a sentence is analytic for the native speakers of a language if they learn its truth in the very process of learning how to use the words that occur in it; and (ii) “recondite” sentences should still count as analytic if they can be obtained by “a chain of inferences each of which individually is assured by the learning of the words” [16] (pp. 79–80). In this perspective, logical truths may qualify as analytic in the traditional sense, although the very existence of enduring disagreement on some logical laws—e.g., on the law of excluded middle on the part of intuitionists—may suggest that such laws are not similarly bound up with the learning of the logical words and “should perhaps be seen as synthetic” [16] (p. 80).

In his latest work Quine appears to leave aside this idea that some logical laws may be synthetic. For example, in his Two dogmas in retrospect, he argues that by the above criterion “all logical truths [...]—that is, the logic of truth functions, quantification, and identity—would then perhaps qualify as analytic, in view of Gödel’s completeness proof” [15] (p. 270) and later on, in a 1993 interview, seems to abandon any hesitation and make his position crystal-clear:

Yes so, on this score I think of the truths of logic as analytic in the traditional sense of the word, that is to say true by virtue of the meaning of the words. Or as I would prefer to put it: they are learned or can be learned in the process of learning to use the words themselves, and involve nothing more [17] (p. 199). (Quoted in [18].)

2.3. Semantic Information

At the half of the 20th century, Bar-Hillel and Carnap’s theory of “semantic information” provided what is, to date, the strongest theoretical justification for the thesis that deductive reasoning is “tautological”. Although their effort was clearly inspired by the rising enthusiasm for Shannon and Weaver’s new Theory of Information [19], their starting point was their dissatisfaction with the nonchalant tendency of fellows scientists to apply its concepts and results well beyond the “warranted areas”. Shannon and Weaver’s central problem was only how uninterpreted data can be efficiently
encoded and transmitted. So the idea of applying their theory to contexts in which the interpretation of data plays an essential role was a major source of confusion and misunderstandings:

The Mathematical Theory of Communication, often referred to also as “Theory of (Transmission of) Information”, as practised nowadays, is not interested in the content of the symbols whose information it measures. The measures, as defined, for instance, by Shannon, have nothing to do with what these symbols symbolise, but only with the frequency of their occurrence. [...] This deliberate restriction of the scope of the Statistical Communication Theory was of great heuristic value and enabled this theory to reach important results in a short time. Unfortunately, however, it often turned out that impatient scientists in various fields applied the terminology and the theorems of Communication Theory to fields in which the term “information” was used, presystematically, in a semantic sense, that is, one involving contents or designata of symbols, or even in a pragmatic sense, that is, one involving the users of these symbols [20].

By way of contrast, they put forward a Theory of Semantic Information, in which “the contents of symbols” were “decisively involved in the definition of the basic concepts” and “an application of these concepts and of the theorems concerning them to fields involving semantics thereby warranted” [20] (p. 148). The basic idea is simple and can be briefly explained as follows.

Suppose we are interested in the weather forecast for tomorrow and that we focus only on the possible truth values of the two sentences “tomorrow will rain” ($R$) and “tomorrow will be windy” ($W$). Then, there are four possible relevant states of the world, described by the following conjunctions:

$$R \land W, \quad R \land \neg W, \quad \neg R \land W, \quad \neg R \land \neg W$$

Now, the sentence “tomorrow will rain and will be windy” is intuitively more informative than the sentence “tomorrow will rain”. We can explain this by noticing that it excludes more possibilities, i.e., more possible (relevant) states of the world. On the other hand, the sentence “tomorrow will rain or will not rain” conveys no information, since it does not exclude any possible state. So, it seems natural to identify the information conveyed by a sentence with the set of all “possible worlds” that are excluded by it, and to assume that its measure should be somehow related to the size of this set.

The same basic idea, identifying the information carried by a sentence with the set of the possible states that it excludes, had already made its appearance in Popper’s *Logic of Scientific Discovery* (1934), where it played a crucial role in defining the “empirical content” of a theory and in supporting Popper’s central claim, namely that the most interesting scientific theories are those that are highly falsifiable, while unfalsifiable theories are devoid of any empirical content:

The amount of positive information about the world which is conveyed by a scientific statement is the greater the more likely it is to clash, because of its logical character, with possible singular statements. (Not for nothing do we call the laws of nature “laws”: the more they prohibit the more they say.) [21] (p. 19).

[...]
by experience; thus compared with the second theory, the first theory may be said to be “falsifiable in a higher degree”. This also means that the first theory says more about the world of experience than the second theory, for it rules out a larger class of basic statements. [...] Thus it can be said that the amount of empirical information conveyed by a theory, or its empirical content, increases with its degree of falsifiability [21] (p. 96).

3. The Anomalies of the Received View

A straightforward consequence of Bar-Hillel and Carnap’s notion of “semantic information” is that contradictions, like “tomorrow will rain and will not rain”, carry the maximum amount of information, since they exclude all possible states. Another inevitable consequence of the theory is that all logical truths are equally uninformative (they exclude no possible world), which justifies their being labelled as “tautologies”. But in classical logic a sentence \( \varphi \) is deducible from a finite set of premises \( \psi_1, \ldots, \psi_n \) if and only if the conditional \( (\psi_1 \land \ldots \land \psi_n) \rightarrow \varphi \) is a tautology. Accordingly, since tautologies carry no information at all, no logical inference can yield an increase of information. Therefore, if we identify the semantic information carried by a sentence with the set of all possible worlds it excludes, we must also accept the inevitable consequence that, in any valid deduction, the information carried by the conclusion is contained in the information carried by the (conjunction of) the premises. While this theory seems to justify the (fourth?) empiricist dogma discussed in Section 2.1, both these consequences appear to be at odds with our intuitions and clash with the commonsense notion of information, to the extent that some authors have described them as true “paradoxes”.

3.1. The Bar-Hillel–Carnap Paradox

Bar-Hillel and Carnap were well aware that their theory of semantic information sounded counterintuitive in connection with contradictory (sets of) sentences, as shown by the near-apologetic remark they included in their [22]:

It might perhaps, at first, seem strange that a self-contradictory sentence, hence one which no ideal receiver would accept, is regarded as carrying with it the most inclusive information. It should, however, be emphasized that semantic information is here not meant as implying truth. A false sentence which happens to say much is thereby highly informative in our sense. Whether the information it carries is true or false, scientifically valuable or not, and so forth, does not concern us. A self-contradictory sentence asserts too much; it is too informative to be true [22] (p. 229).

Popper had also realized that his closely related notion of empirical content worked reasonably well only for consistent theories, since all basic statements are potential falsifiers of all inconsistent theories, which would therefore, without this requirement, turn out to be the most scientific of all. So, for him, “the requirement of consistency plays a special role among the various requirements which a theoretical system, or an axiomatic system, must satisfy” and “can be regarded as the first of the requirements to be satisfied by every theoretical system, be it empirical or non-empirical” [21] (p. 72). So, “whilst tautologies, purely existential statements and other nonfalsifiable statements assert, as it were, too little about the class of possible basic statements, self-contradictory statements assert too much. From a
self-contradictory statement, any statement whatsoever can be validly deduced” [21] (p. 71). In fact, what Popper claimed was that the information content of inconsistent theories is null, and so his definition of empirical information content as monotonically related to the set of potential falsifiers was intended only for consistent ones:

But the importance of the requirement of consistency will be appreciated if one realizes that a self-contradictory system is uninformative. It is so because any conclusion we please can be derived from it. Thus no statement is singled out, either as incompatible or as derivable, since all are derivable. A consistent system, on the other hand, divides the set of all possible statements into two: those which it contradicts and those with which it is compatible. (Among the latter are the conclusions which can be derived from it.) This is why consistency is the most general requirement for a system, whether empirical or non-empirical, if it is to be of any use at all [21] (p. 72).

3.2. The Scandal of Deduction

Cohen and Nagel were among the first to point out that the traditional tenet that logical deduction is devoid of any informational content sounds paradoxical:

If in an inference the conclusion is not contained in the premises, it cannot be valid; and if the conclusion is not different from the premises, it is useless; but the conclusion cannot be contained in the premises and also possess novelty; hence inferences cannot be both valid and useful [23] (p. 173).

A few decades later Jaakko Hintikka described this paradox as a true “scandal of deduction”:

C.D. Broad has called the unsolved problems concerning induction a scandal of philosophy. It seems to me that in addition to this scandal of induction there is an equally disquieting scandal of deduction. Its urgency can be brought home to each of us by any clever freshman who asks, upon being told that deductive reasoning is “tautological” or “analytical” and that logical truths have no “empirical content” and cannot be used to make “factual assertions”: in what other sense, then, does deductive reasoning give us new information? Is it not perfectly obvious there is some such sense, for what point would there otherwise be to logic and mathematics? [5] (p. 222).

The standard answer to this question has a strong psychologistic flavour. According to Hempel: “a mathematical theorem, such as the Pythagorean theorem in geometry, asserts nothing that is objectively or theoretically new as compared with the postulates from which it is derived, although its content may well be psychologically new in the sense that we were not aware of its being implicitly contained in the postulates” ([11] (p. 9), Hempel’s emphasis.) This implies that there is no objective (non-psychological) sense in which deductive inference yield new information. Hintikka’s reaction to this typical neopositivistic way out of the paradox is worth quoting in full:

If no objective, non-psychological increase of information takes place in deduction, all that is involved is merely psychological conditioning, some sort of intellectual psychoanalysis,
calculated to bring us to see better and without inhibitions what objectively speaking is already before your eyes. Now most philosophers have not taken to the idea that philosophical activity is a species of brainwashing. They are scarcely any more favourably disposed towards the much more far-fetched idea that all the multifarious activities of a contemporary logician or mathematician that hinge on deductive inference are as many therapeutic exercises calculated to ease the psychological blocks and mental cramps that initially prevented us from being, in the words of one of these candid positivists, “aware of all that we implicitly asserted” already in the premises of the deductive inference in question [5] (pp. 222–223).

3.3. Wittgenstein and the “Perfect Notation”

A non-psychologistic attempt to avoid the paradox consists in blaming it on the imperfection of our logical language. In his *Tractatus*, Wittgenstein raises the question of an “adequate notation” through which each sentence shows its meaning, where the latter is to be identified with the possibility of its being true or false: “The sense of a proposition is its agreement and disagreement with the possibilities of the existence and non-existence of the atomic facts.” (T. 4.2). While the truth of an elementary proposition consists in the existence or non-existence of a certain fact about the world, the truth of complex propositions depends on the logical relations between the elementary propositions occurring in them: complex propositions are truth functions of the elementary propositions. Thus, the meaning of a proposition consists in the conditions under which it is true or false, and an adequate notation should be able to show these conditions explicitly: “a proposition shows its sense” (T. 4.022). Nevertheless, “[in common language] it is humanly impossible to deduce the logic of language” (T. 4.002), because the grammatical structure does not mirror the logical structure of the sentence itself. The logic underlying linguistic utterances could instead be made evident by a more appropriate symbolism, one capable of making it immediately visible without resorting to any “deductive process”.

In a logically perfect language the recognition of tautologies should be immediate. Since the deducibility of a certain conclusion from a given set of premises is equivalent to the tautologyhood of the conditional whose antecedent is the conjunction of the premises and whose consequent is the conclusion of the inference, the correctness of any inference would prove, in a symbolism of the kind, to be immediately visible. So, given a “suitable notation”, logical deduction could actually be reduced to the mere inspection of propositions:

When the truth of one proposition follows from the truth of others, we can see this from the structure of the propositions. (*Tractatus*, 5.13)

In a suitable notation we can in fact recognize the formal properties of propositions by mere inspection of the propositions themselves. (6.122).

Every tautology itself shows that it is a tautology. (6.127(b))

In accordance with Wittgenstein’s idea, one could specify a procedure that translates sentences into a “perfect notation” that fully brings out the information they convey, for instance by computing the whole truth-table for the conditional that represents the inference. Such a table displays all the relevant possible
worlds and allows one to distinguish immediately those that make a sentence true from those that make it false, the latter representing (collectively) the “semantic information” carried by the sentence. Once the translation has been performed, logical consequence can be recognized by “mere inspection”.

Thus, if information could be fully unfolded by means of some mechanical translation into a “perfect logical language”, the scandal of deduction could be avoided without appealing to psychologism. Sometimes we fail to immediately “see” that a conclusion is implicit in the premises because we express both in a concise notation, a sort of stenography that prevents us from fully recognizing the formal properties of propositions until we decode it into an adequate notation. From this point of view, semantic information would be a perfectly good way of specifying the information carried by a sentence with reference to an algorithmic procedure of translation. (On the theme of a “logically perfect language” see also [24].)

3.4. Hintikka on the Scandal of Deduction

Although this idea may seem to work well for propositional logic, one can easily see how the Church–Turing undecidability theorem excludes the possibility of a perfect language, in Wittgenstein’s sense, for first-order logic: since first-order logical truth is undecidable, we can never find an algorithm to translate every sentence into a perfect language in which its tautologyhood could be immediately decided by mere inspection. This negative result is also the main motivation for Hintikka’s criticism of Bar-Hillel and Carnap’s notion of semantic information.

… measures of information which are not effectively calculable are well-nigh absurd. What realistic use can there be for measures of information which are such that we in principle cannot always know (and cannot have a method of finding out) how much information we possess? One of the purposes the concept of information is calculated to serve is surely to enable us to review what we know (have information about) and what we do not know. Such a review is in principle impossible, however, if our measures of information are non-recursive [5] (p. 228).

Hintikka’s positive proposal consists in distinguishing between two objective and non-psychological notions of information content: “surface information”, which may be increased by deductive reasoning, and “depth information” (equivalent to Bar-Hillel and Carnap’s “semantic information”), which may not. While the latter justifies the traditional claim that logical reasoning is tautological, the former vindicates the intuition underlying the opposite claim. In his view, first-order deductive reasoning may increase surface information, although it never increases depth information (the increase being related to deductive steps that introduce new individuals). Without going into details (for a criticism of Hintikka’s approach see [25]), we observe here that Hintikka’s proposal classifies as non-analytic only some inferences of the non-monadic predicate calculus and leaves the “scandal of deduction” unsettled in the domain of propositional logic:

The truths of propositional logic are […] tautologies, they do not carry any new information. Similarly, it is easily seen that in the logically valid inferences of propositional logic the information carried by the conclusion is smaller or at most equal to the information carried
by the premises. The term “tautology” thus characterizes very aptly the truths and inferences of propositional logic. One reason for its one-time appeal to philosophers was undoubtedly its success in this limited area” ([5] (p. 154)).

Hence, in Hintikka’s view, for every finite set of Boolean sentences \( \Gamma \) and every Boolean sentence \( \varphi \),

If \( \Gamma \vdash \varphi \) the information carried by \( \varphi \) is included in the information carried by \( \Gamma \) \hspace{1cm} (1)

This is highly unsatisfactory, especially since the theory of computational complexity has revealed that the decision problem for Boolean logic is co-NP-complete [26], that is, among the hardest problems in co-NP. Although not a proved theorem, it is a widely accepted conjecture that Boolean logic is *practically undecidable*, i.e., admits of no feasible decision procedure. (This means that every decision procedure for Boolean logic is bound to be superpolynomial in the worst case. On the other hand, there are very efficient decision algorithms around that work quite efficiently on average. In [27] Finger and Reis present a very interesting empirical analysis of the runtime distribution of a variety of decision methods on randomly generated formulas.) To express the same idea in a different way, we could say that there cannot be any “perfect” propositional language, in Wittgenstein’s sense—one in which the logical relations between sentences can be recognized by mere inspection of the sentences themselves—into which a conventional logical language can be feasibly translated. (On this point see [24].)

Thus, some degree of uncertainty about whether or not a certain conclusion follows from given premises cannot be, in general, completely eliminated even in the restricted and “simple” domain of propositional logic. So, if we take seriously the time-honoured and common-sense concept of information, according to which information consists in reducing uncertainty, we should conclude that in some cases deductive reasoning *does* reduce our uncertainty, and therefore increases our information, even at the propositional level.

The scandal of deduction has recently received renewed attention leading to a number of original contributions (e.g., [28] (Chapter 2), [1,25,29–31] that do not appear, however, to be reducible to a single conceptual paradigm.

3.5. The Problem of Logical Omniscience

Another widely debated paradox connected with the received view on logical deduction arises in the context of modal characterizations of propositional attitudes and is nothing but a variant of the “scandal of deduction” described in the previous section. According to the standard logic of knowledge (epistemic logic) and belief (doxastic logic), as well as to the more recent attempts to axiomatize the “logic of being informed” (information logic), if an agent \( a \) knows (or believes, or is informed) that a sentence \( \varphi \) is true, and \( \psi \) is a logical consequence of \( \varphi \), then \( a \) is supposed to know (or believe, or be informed) also that \( B \) is true. (For a survey on epistemic and doxastic logic see [32,33]; for information logic, or “the logic of being informed”, see [34,35].) This is often described as paradoxical and labelled as “the problem of logical omniscience”. Let \( \square_a \) express any of the propositional attitudes at issue, referred to the agent \( a \). Then, the “logical omniscience” assumption can be expressed by saying that, for any finite set \( \Gamma \) of sentences,

if \( \square_a \varphi \) for all \( \varphi \in \Gamma \) and \( \Gamma \vdash \psi \), then \( \square_a \psi \) \hspace{1cm} (2)
where $\vdash$ stands for the relation of logical consequence. Observe that, letting $\Gamma = \emptyset$, it immediately follows from (2) that any rational agent $a$ is supposed to be aware of the truth of all classical tautologies, that is, of all the sentences of a standard logical language that are “consequences of the empty set of assumptions”. In most axiomatic systems of epistemic, doxastic and information logic assumption (2) emerges from the combined effect of the “distribution axiom”, namely,

$$\Box_a(\varphi \rightarrow \psi) \rightarrow (\Box_a \varphi \rightarrow \Box_a \psi)$$

and the “necessitation rule”:

$$\text{(N)} \text{ if } \vdash \varphi, \text{ then } \vdash \Box_a \varphi.$$ 

On the other hand, despite its paradoxical flavour, (2) seems an inescapable consequence of the standard Kripke-style semantical characterization of the logics under consideration. The latter is carried out in terms of structures of the form $(S, \tau, R_1, \ldots, R_n)$, where $S$ is a set of possible worlds, $\tau$ is a function that associates with each possible world $s$ an assignment $\tau(s)$ of one of the two truth values (0 and 1) to each atomic sentence of the language, and each $R_a$ is the “accessibility” relation for the agent $a$. Intuitively, if $s$ is the actual world and $sR_at$, then $t$ is a world that $a$ would regard as a “possible” alternative to the actual one, i.e., compatible with what $a$ knows (or believes, or is informed of). Then, the truth of complex sentences is defined, starting from the initial assignment $\tau$, via a forcing relation $|=$. This incorporates the usual semantics of classical propositional logic and defines the truth of $\Box_a \varphi$ as “$\varphi$ is true in all the worlds that $a$ regards as possible”. In this framework, given that the notion of truth in a possible world is an extension to the modal language of the classical truth-conditional semantics for the standard logical operators, (2) appears to be both compelling and, at the same time, counter-intuitive.

Now, under this reading of the consequence relation $\vdash$, which is based on classical propositional logic, (2) may perhaps be satisfied by an “idealized reasoner”, in some sense to be made more precise, but is not satisfied, and is not likely to ever be satisfiable, in practice. (It should be noted that the appeal to an “idealized reasoner” has usually the effect of sweeping under the rug a good deal of interesting questions, including how idealized such a reasoner should be. Idealization may well be a matter of degree.) As mentioned above, even restricting ourselves to the domain of propositional logic, the theory of computational complexity tells us that the decision problem for Boolean logic is co-NP-complete, and this means that any real agent, even if equipped with an up-to-date computer running a decision procedure for Boolean logic, will never be able to feasibly recognize that certain Boolean sentences logically follow from sentences that she regards as true. So, the clash between (2) and the classical notion of logical consequence, which arises in any real application context, may only be solved either by waiving the assumption stated in (2), or by waiving the consequence relation of classical logic in favour of a weaker one with respect to which it may be safely assumed that the modality $\Box_a$ is closed under logical consequence for any practical reasoner.

4. Floridi on the Received View

In this section we discuss Floridi’s ideas on the anomalies of the received view. The reader is warned that our exposition shows a strong bias for the ideas put forward in a joint paper by Floridi and the present author [1]. However, we hope that, as often is the case, this bias may have a heuristic value.