

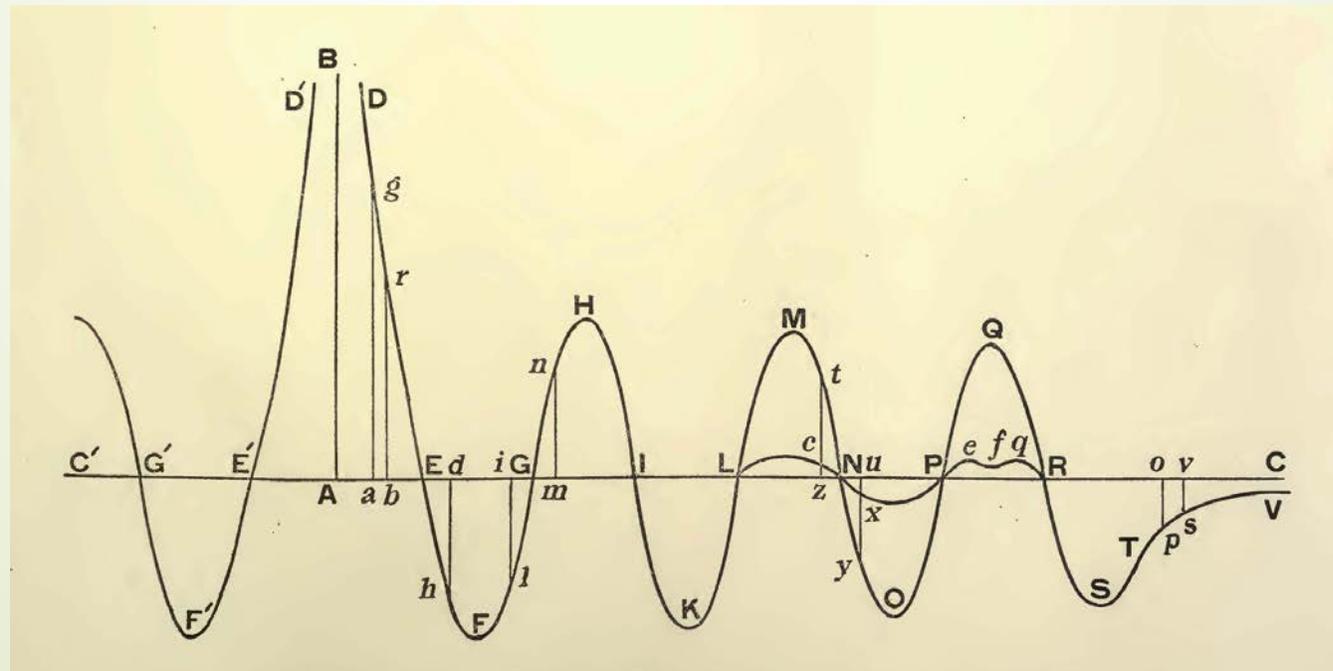
Ragusa 1711 - Milan 1787

Geometrizing world images

How Geometry Shaped R.J. Boscovich's Natural Philosophy

Boscovich and the Forces

- Boscovich's curve** (*Philosophiae naturalis theoria redacta ad unicum legem virium in natura existentium*, prostat Viennae Austriae, in officina libraria Kaliwodiana, 1758; 2° edition: *Theoria philosophiae naturalis redacta ad unicum legem virium in natura existentium*, Editio veneta prima, Venetiis, Ex Typographia Remondiniana, 1763).



A dynamical theory of matter?

- ▶ M. Friedman, Introduction to I. Kant, *Metaphysical Foundations of Natural Science*, p. x
 - ▶ Dynamism is a conception in which “the basic properties of solidity and impenetrability are not taken as primitive and self-explanatory, but are rather viewed as derived from an interplay of forces — here, more specifically, the two fundamental forces of attraction and repulsion, which together determine a limit or boundary beyond which repulsion (and thus impenetrability) is no longer effective and attraction (representing Newtonian gravitation) then takes over unhindered. [...It] exerted a powerful influence in the later part of the eighteenth century, in the work of such thinkers as Boscovich and Priestley, for example.”
- ▶ M. Jammer, *Concepts of force* (1957, p. 148):
 - ▶ “Newtonian physics, together with Leibnizian monadology, formed the basis also of a different school of thought, perhaps best characterized by Rogerius Josephus Boscovich, for whom force was the ultimate element of reality.”

Boscovich and Newton(ianism) /1

- ▶ Newton's idea (which Boscovich was well aware of):
 - ▶ "Et sicuti in Algebra, ubi Quantitates affirmativæ evanescent & desinunt, ibi negativæ incipiunt; ita in Mechanicis, ubi Attractio desinit, ibi Vis repellens succedere debet." *Optice* (1706), Quaestio 23, p. 388 (see *Opticks*, 1730, Query 31, p. 395: "As in Algebra, where affirmative Quantities vanish and cease, there negative ones begin; so in Mechanicks, where Attraction ceases, there a repulsive Virtue ought to succeed").
- ▶ First generation of Newtonians (Keill, Freind, 'sGravesande, Pemberton) mostly concerned with attractions ("repulsive virtue" sparingly mentioned).
 - ▶ Attractions are troubling enough: more than one fundamental force!
 - ▶ How could different forces interact? What are their mutual relations, if any?
 - ▶ Solution through investigation of the microscopic conditions of matter.
- ▶ The problem set out in the *Opticks*: clarifying the microscopic actions of the force(s), i.e. the constitution of matter.
 - ▶ In the *Queries* Newton exposed in a highly speculative way a theory of matter!

Boscovich and Newton(ianism) /2

- ▶ Other Newtonians (Hales, Desaguliers, and later Gowin Knight), considered also repulsive force and gave it the same rank of attraction (see Heilbron 1979).
 - ▶ Newton had considered air consisting of repelling particles whose force acts inversely as the distance; he also discussed chemical dissociation on such basis.
 - ▶ Hales made measurements of the amount of "air" in organic and inorganic substances and supposed (1727) that air may be in either an attractive or a repulsive state, but not in both simultaneously.
 - ▶ **Desaguliers extends the theory: matter particles simultaneously attract (cohesion) and repel (elasticity) one another. Matter is homogeneous. But "particles are at the centers of alternating spheres of attractive and repulsive forces" and their approaching or receding from one another is an accident of distance.**
 - ▶ G. Knight, *An attempt to demonstrate that all the phenomena in nature may be explained by two simple active principles, attraction and repulsion* (1754)

Newtonians' Program and Boscovich's

- ▶ Boscovich is a truly Newtonian because of the problems involved and the style "attracting/repelling forces" due to distances.
- ▶ What is really new is Boscovich's application of Newton's original idea of the inversion of forces, which leads him to a remarkable result.

Newtonians:

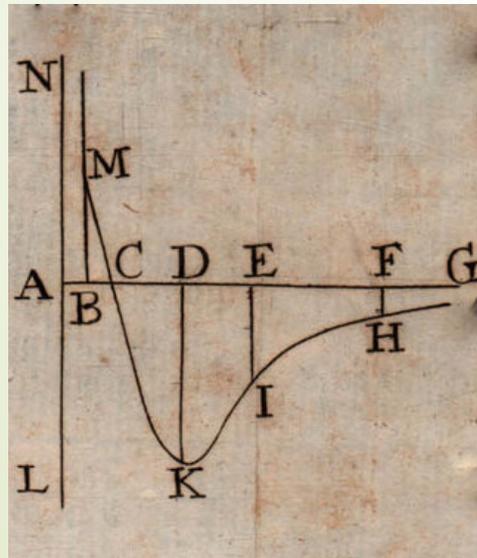
0 distance = strongest force = cohesion

Boscovich:

cohesion = 0 force = x distance

Newtonians' Program and Boscovich's

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"The Parts of all homogeneous hard Bodies which fully touch one another, stick together [*cohaerent*] very strongly [...]. I had infer from their Cohesion, that their Particles attract one another by some Force, which in immediate Contact is exceeding strong." (Opticks, Q31, pp. 364-365.)

A dynamical theory of matter?

- ▶ M. Friedman, Introduction to I. Kant, *Metaphysical Foundations of Natural Science*, p. x
 - ▶ This is a theory “according to which the basic properties of solidity and impenetrability are not taken as primitive and self-explanatory, but are rather viewed as derived from an interplay of forces — here, more specifically, the two fundamental forces of attraction and repulsion, which together determine a limit or boundary beyond which repulsion (and thus impenetrability) is no longer effective and attraction (representing Newtonian gravitation) then takes over unhindered. [...It] exerted a powerful influence in the later part of the eighteenth century, in the work of such thinkers as Boscovich and Priestley, for example.”
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A career in mathematics, both pure and mixed

- ▶ 1741/42: Professor of Mathematics at the (Jesuits') Collegium Romanum
- ▶ 1763/64: Professor of Mathematics at the University of Pavia
- ▶ 1769: Professor of Astronomy and Optics in Milan (Scuole Palatine)
- ▶ 1772/73: He resigns his post at the Observatory and the Scuole Palatine and goes to France
 - ▶ 1773: the Society of Jesus is suppressed (*bull*a «*Dominus ac Redemptor*», 1773 July 21, signed by Clement XIV)
- ▶ 1773: For him was created the post of «Director of Naval Optics»
- ▶ 1782: In Italy again, just to attend the print of his works in 5 volumes: *Opera pertinentia ad Opticam et Astronomiam* (1785).
 - ▶ (he didn't come back to France)
- ▶ 1785: in Milan, invited by the Absburg plenipotentiary Johann von Wilczek
- ▶ 1787: death

What is mixed mathematics?

*Cattedra / triennale /
di Matematiche applicate
per le Scuole Palatine.*

*Quando la Geometria, e l'Algebra
sortono dal mondo intellettuale
e s'immergono più addentro nella
materia, considerando i rapporti
particolari delle grandezze sensi-
bili, i moti per esempio degli astri,
l'aumento delle forze moventi,
le vie della luce ne varj mezzi,
gli effetti del suono per le vibra-
zioni delle corde, allora acquisi-
tano il nome di Matematiche
Miste o Applicate.*

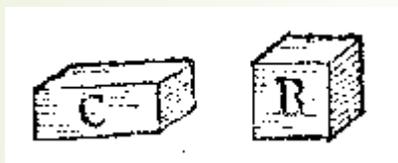
/Three-year/ Chair
of Applied Mathematics
for the Palatin Schools

When Geometry and Algebra leave the intellectual world and dive deeper into the matter, considering the peculiar relations of the sensible magnitudes, e.g. the aster motions, the growth of motive forces, the ways taken by the light in various mediums, the effects of the sound because of chord vibrations, then they gain the name of Mixed or Applied Mathematics.

Boscovich's publications until 1745

- ▶ Until the 40s: astronomical (observational or practical) papers; some works in mixed mathematics or mixed mathematics-oriented, some works in *physica particularis* (aurora borealis, mathematical argument for the sphericity of the Earth, difference of gravity on the Earth surface, osculating circle, etc...)
- ▶ *Trigonometriae sphaericae constructio*, 1737
- ▶ *De circulis osculatoribus dissertatio*, 1740
- ▶ *De motu corporum projectorum in spatio non resistente*, 1740
- ▶ *De natura et usu infinitorum et infinite parvorum*, 1741
- ▶ Damage survey on the dome of St. Peter's Basilica in Rome, 1742
- ▶ *De cycloide et logistica*, 1743
- ▶ *Trigonometria Sphaerica*
- ▶ *De viribus vivis*, 1745

Generatio velocitatis: Descartes' first rule of collision



- «First, if these two bodies, say B and C, were wholly equal and were moved equally fast, B from the right toward the left and C on a line with it [*illi in directum*] from the left toward the right, when they collided with one another, they would be reflected and afterward would continue to be moved, B toward the right and C toward the left, no part of their speed having been lost.»

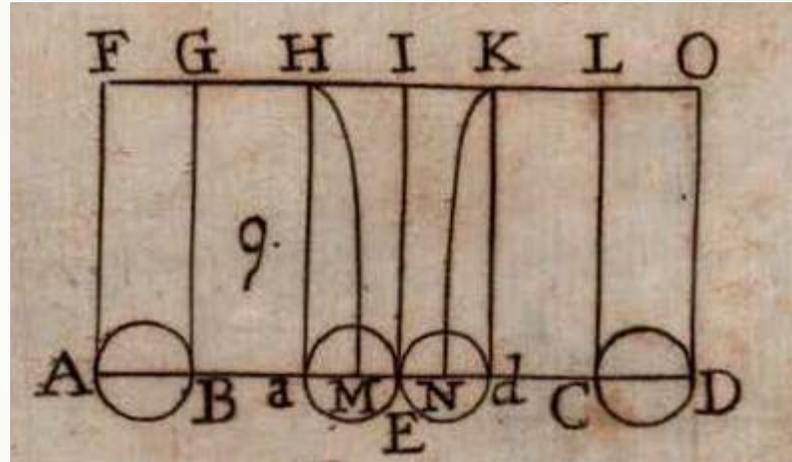
(R. Descartes, *Principles of Philosophy*, Part II, «On Motion», § 46)

[example: <http://vam.anest.ufl.edu/physics/collisionphysics.html>]

Generatio velocitatis: Boscovich criticism of Leibnizians' argument

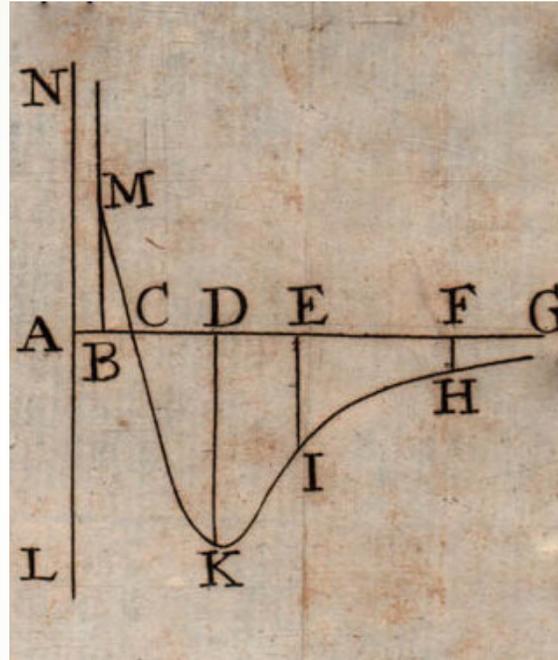
- ▶ Bodies change velocity abruptly (Cartesian)
- ▶ Bodies don't change velocity abruptly, because there are no absolutely hard bodies (Leibnizian)
- ▶ Bodies cannot change velocity abruptly (it is *a priori* excluded), **but** a sudden change in velocity seems to be not avoidable by the Leibnizians' assumption of only 'flexible' bodies.
 - ▶ **Boscovich's question:** How is possible to avoid jumps in velocity (in magnitudes)?

Generatio velocitatis: Boscovich criticism of Leibnizians' argument



Leibnizians' mistake according to Boscovich: FA, BG... LC, OD etc. represent (equal) velocity of the particles forming two (equal) colliding spheres in E. Also in the case of flexible, soft bodies, in the instant of collision all velocities diminish gradually (like HM, KN), except the velocities of the particles/points B and C, which abruptly become = 0 (as if IE disappears)

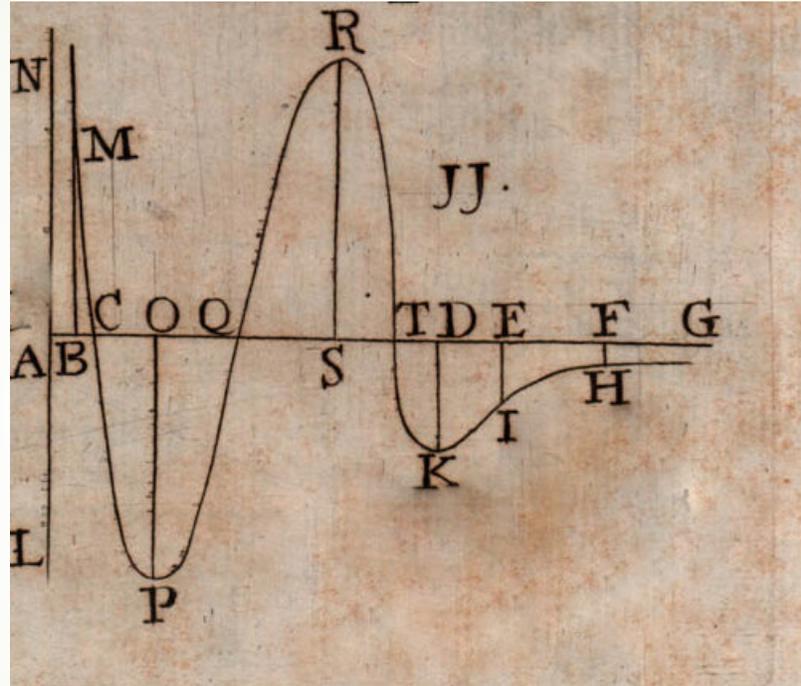
The first appearance of the Curve



De viribus vivis, 1745

- ▶ Bodies must slow down when approaching one another – to account for this, a repulsive force must be acting at smallest distances: as bodies move nearer, and come very near, attraction decreases and the force ends up in a repulsion which slows them down and finally pushes them back until a point of equilibrium, where repulsion is balanced out by attraction (let's say in C, that is a zero on the x-axis).
 - ▶ Consequence: contact doesn't exist!

Geometrical properties in foreground



De viribus vivis, 1745

- an a priori assumption (metaphysical statement): there are no jumps in magnitudes in nature → continuity
- geometrical-mathematical approach
 - No physical assumptions about forces
 - Properties of forces depend on geometrical-mathematical properties

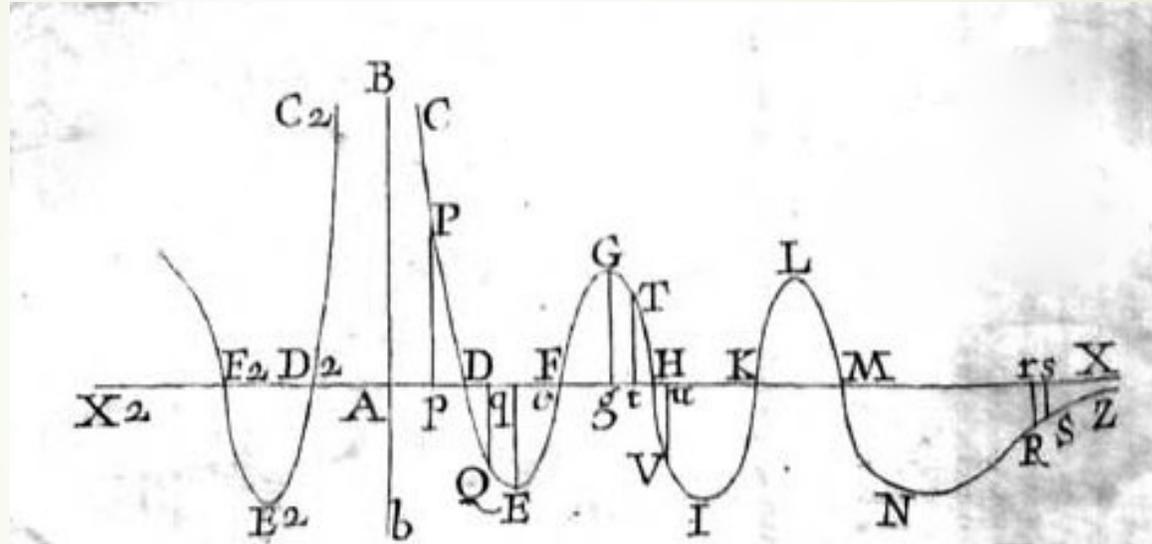
Geometrical properties in foreground

As *potentiae* are only meant those causes which, through their own actions, determine a body to change its state [...]. Wherever such *potentiae* should be physically posed, as far as they cause in a body the same change in velocity (*velocitas actu primo*), that is determine it to assume the same instantaneous velocity (*velocitas actu secundo*), the same phenomena of motion will occur. (*De viribus vivis*, 1745, 45)

The name of attractive and repulsive force is "an arbitrary designation, even if habitual and not unfitting" [...]. (*De lege virium in natura existentium*, 1755, n. 43)

- ▶ an a priori assumption (metaphysical statement): there are no jumps in magnitudes in nature → continuity
- ▶ geometrical-mathematical approach
 - ▶ **No physical assumptions about forces**
 - ▶ **Properties of forces depend on geometrical-mathematical properties**

Forming new forms



- ▶ *Dissertatio de lumine, pars secunda, 1748*
 - ▶ A negative branch appears – does it have any physical meaning?
 - ▶ «To that asymptote must approach the two legs DC tending to repulsive parts. They must be absolutely of the same look as well as equal, and the area $BADC$ enclosed by the axis, the asymptote, and the asymptotic leg must be infinite».

Boscovich's papers on natural philosophy, 1745-1763

Big deal of pure mathematical work, mainly on curves, announced in *De maris aestu*, 1747

- ▶ *De viribus vivis*, 1745
 - ▶ **First form of the curve**
- ▶ *Dissertatio de lumine, pars secunda*, 1748
 - ▶ **Second form of the curve**
- ▶ *De materiae divisibilitate et principiis corporum*, composed '48, published '57
- ▶ *De continuitatis lege*, 1754
- ▶ *De lege virium in natura existentium*, 1755
 - ▶ **Final form of the curve, and analytical expression**
- ▶ *Philosophiae naturalis theoria*, 1758
- ▶ *Theoria philosophiae naturalis*, 1763 (with some additions)

Publication of the Handbook of Geometry and Mathematics, 1752-1754 (3 volumes + 1 treatise added to the **3° vol., on conic sections**: *De transformatione locorum geometricorum, ubi de continuitatis lege, ac de quibusdam Infiniti mysteriis*, 1754)

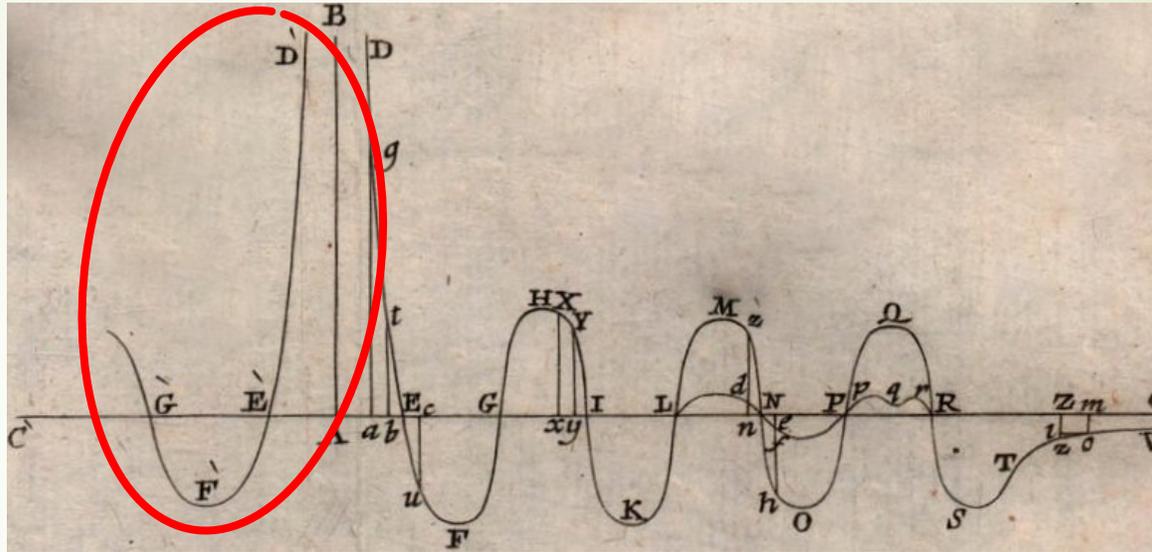
What does negativity mean /1

- ▶ Some useful indication from the way in which Boscovich was developing his mathematics at the end of the 1740s / beginning of the 1750s:
 - ▶ «In the second part of my dissertation *De lumine* all my theory is sketched from the beginning. In the work [he is speaking of a comprehensive work] there will be a little more of substance as far as physics is concerned, even if there will be **much more on metaphysics, geometry, and Calculus**» (Letter to Natale Boscovich, 1748 september 14th).
 - ▶ In announcing a volume of his *Elementa matheseos* (his handbook for students at the Collegium Romanum) in a work *De maris aestu* (1747, *On the sea-tides*): «The straight line for its own nature runs on both sides to the infinity and has no limits. **Who assumes a bounded straight line, assumes a segment of a locus geometricus**, in the same way in which one who assumes an arc of a circle, assumes a segment of the circumference. Besides, **the straight line itself is, in a certain manner, like a circumference of an infinite circle, which at that infinite distance in a certain manner comes back in itself, as if it joins with itself**. This applies to the legs of a parabola as well as to the opposite branches of a hyperbola, but holds true for the infinite legs of any *locus geometricus* too [...]».

What does negativity mean /2

- ▶ This kind of proto-projective geometry (probably independent of Desargues's) is mainly developed in a long treatise added to the 3^o vol. of the *Elementa matheseos*: ***De transformatione locorum geometricorum, ubi de continuitatis lege, ac de quibusdam Infiniti mysteriis (1754)***.
 - ▶ Central idea: loci geometrici having a 'leg' projected to the infinity come back from the infinity and re-emerge from the other ('negative') side of the axis (with more possibilities).
 - ▶ Negative magnitudes don't really exist: they are not 'real'. They are relative ('debits')
 - ▶ Negative magnitudes are «plus quam infinitae»
 - ▶ You have just one kind of ∞ ; you don't have $-\infty$
 - ▶ Loci geometrici are **continuous**, never interrupted, never breakable lines
 - ▶ 1754 is the same years of *De continuitatis lege*
 - ▶ This philosophy of mathematics is developed further
 - ▶ The law of continuity is explicitly said to be the *a priori* fundament of the law of forces

Why drawing a curve without meaning



Why drawing it, if we can —or even must— abstract from it?

Boscovich's commitment to mathematics!

- It compels him to take continuity into account;
- and prevent him to destroy continuity while operating on a given curve

The virtue of mathematics

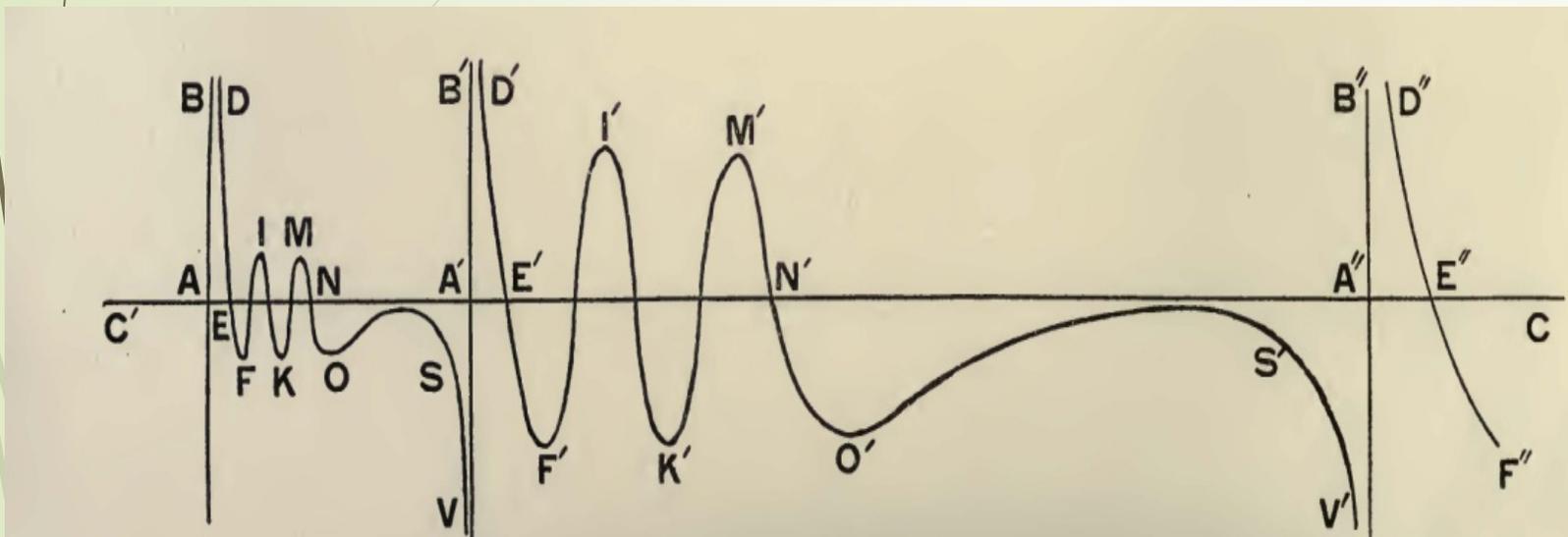
Difficultas residua
sublata ab hac
Theoria.

405. Adhuc tamen, quoniam nostra vita, & memoria respectu immensi fortasse subsecuturi ævi est itidem fere nihil; [185] si gravitas generalis in infinitum protendatur cum eadem illa lege, & eodem asymptotico crure, utique non solum hoc systema nostrum solare, sed universa corporea natura ita, paulatim utique, sed tamen perpetuo ab eo statu recederet, in quo est condita, & universa ad interitum necessariò rueret, ac omnis materia deberet demum in unicum informem massam conglobari, cum fixarum gravitas in se invicem, nullo obliquo, & curvilineo motu elidatur. Id quidem haud ita se habere, demonstrari omnino non potest; adhuc tamen Divinæ Providentiæ videtur melius consulere Theoria, quæ ejus etiam ruinæ universalis evitandæ viam aperiat, ut aperit sane mea. Fieri enim potest, uti notavimus n. 170, ut postremus ille curvæ meæ arcus, qui exhibet gravitatem, posteaquam recesserit ad distantias majores, quam sint cometarum omnium ad nostrum solare systema pertinentium distantia maximæ a Sole, incipiat recedere plurimum ab hyperbola habente ordinatas reciprocas quadratorum distantia, ac iterum axem secet, & contorqueatur. Eo pacto posset totum aggregatum fixarum cum Sole esse unica particula ordinis superioris ad eas, quæ hoc ipsum systema componunt, & pertinere ad systema adhuc in immensum majus & fieri posset ut plurimi sint ejus generis ordines particularum ejusmodi etiam, ut ejusdem ordinis particulæ sint penitus a se invicem segregatæ sine ullo possibili comætu ex una in aliam per asymptoticos arcus plures meæ curvæ juxta ea, quæ exposui a num. 171.

Philosophiæ naturalis theoria, 1758

- Cosmological paradox (Newton-Bentley) solved by mathematical properties of the curve:
 - New intersections of the curve with the axis at interstellar distances admitted
 - Equilibrium granted
- Intermediate asymptotes admitted:
 - The Solar System and the fixed stars form a sub-aggregate – «a particle» – of an aggregate of higher order and contains sub-aggregates of lower orders.
 - Because of the mathematics of the curve, the aggregates are independent

The virtue of mathematics



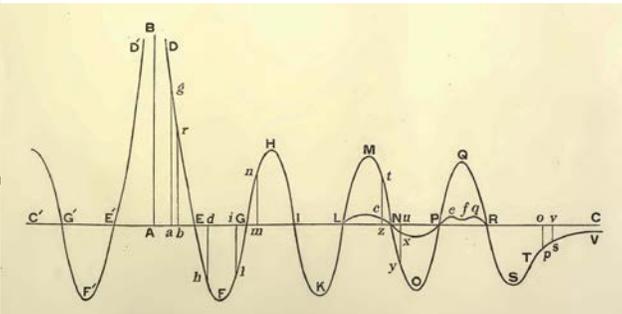
Philosophiæ naturalis theoria, 1758,
fig. 14 to n. 171

The 'standard form' of the curve is nothing but one possibility.

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 - New intersections of the curve with the axis at interstellar distances admitted
 - Equilibrium granted
- Intermediate asymptotes admitted:
 - The Solar System and the fixed stars form a sub-aggregate – «a particle» – of an aggregate of higher order and contains sub-aggregates of lower orders.
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The analytical form

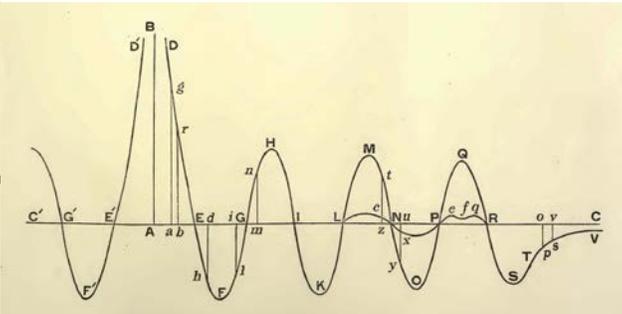
$$y = \frac{z^m + az^{m-1} + bz^{m-2} + cz^{m-3} + \dots + f}{z(z^r + gz^{r-1} + hz^{r-2} + \dots + l)}$$



- The curve is expressed as a convergent series of mathematical terms in powers of the inverse square of the distance ($z = x^2$); the exponent m is the number of values like AE, AG, AI...
- There is a "hidden" dependence from the product of masses (the law is expressed as if points are concerned, and each point has mass=1)

Fundamental theorem of algebra

$$y = \frac{z^m + az^{m-1} + bz^{m-2} + cz^{m-3} + \dots + f}{z(z^r + gz^{r-1} + hz^{r-2} + \dots + l)}$$



- Degree of the equation = numbers of solutions (proved only by Gauss; but known since late 16th century).
- Degree of the equations = numbers of 0s on the x-axis = numbers of inversions of the force direction.
 - That number is an empirical question. Nevertheless, the theorem provides a strong **constraint** to physics.