SESSION VI

NEWTON'S PROGRAMME OF A «RATIONAL MECHANICS» AS THE SCIENCE OF MOTIONS & FORCES

OUTLINE

- Newton compared with Pappus' *Collections*
  - Newton and the ancients
  - Several main themes of Newton are made clear through his relationship with ancients, like…
- The introduction of forces
  - The «twofold» concept of force in Newton
  - Force in mechanics as a (logically) locomotion-dependent concept

NEWTON & PAPPUS

NEWTON'S PREFACE TO THE *PRINCIPIA* (1687)

Since the ancients (according to **Pappus**) considered *mechanics* to be of the greatest importance in the investigation of nature and science and since the moderns – rejecting substantial forms and occult qualities – have undertaken to reduce the phenomena of nature to mathematical laws, it has seemed best in this treatise to concentrate on *mathematics* as it relates to natural philosophy. The ancients divided *mechanics* into two parts: the *rational*, which proceeds rigorously through demonstrations, and the *practical*. *Practical mechanics* is the subject that comprises all the manual arts, from which the subject of *mechanics* as a whole has adopted its name. […] *Rational mechanics* will be the science, expressed in exact propositions and demonstrations, of the motions that result from any forces whatever and of the forces that are required for any motions whatever.

PAPPUS'S *Synagoge* (c. 340)

The science of mechanics […] is not merely useful for many important practical undertakings, but is justly esteemed by philosophers and is diligently pursued by all who are interested in mathematics, since it is fundamentally concerned with the doctrine of nature with special reference to the material composition of the elements in the cosmos. For it examines bodies at rest, their natural tendency, and their locomotion in general, not only assigning causes of natural motion, but devising means of impelling bodies to change their position, contrary to their natures, in a direction away from their natural places. In this the science of mechanics uses theorems suggested to it by a consideration of matter itself. Now the mechanicians of Hero's school tell us that the science of mechanics consists of a theoretical and a practical part. The theoretical part includes geometry, arithmetic, astronomy, and physics, while the practical part consists of metal-working, architecture, carpentry, painting, and the manual activities connected with these arts.

NEWTON'S PERSPECTIVE:

- The Ancients: mechanics important for the investigation of science
    - Pappus: mechanics is concerned with *motion and rest*
        - Studying causes of natural motion
        - Devising means for changing the natural places of bodies
- The Moderns:
    - Reduce natural phenomena to mathematical laws
    - Rejects substantial forms and occult qualities
- Division in practical and rational («theoretical») mechanics
    - Newton: PM embraces all the manual arts (science of machines, as Pappus let suspect)
        - Pappus: metal-working, architecture, carpentry, painting, etc. (science of machines needed in these activities, essentially in order to raise and lower weights)
    - Newton: RM (=TM) as the mathematical (geometrical) science of motions and forces
        - Pappus: TM (=RM) includes geometry, arithmetic, astronomy, and physics (*mathesis mixta*)

But Newton: ontological equivalence between motion and rest!

CAUTIONARY NOTE ON «PURE» AND «MIXED» MATHEMATICS

H.J.M. Bos, “Mathematics and rational mechanics”. In *The Ferment of Knowledge. Studies in the Historiography of Eighteenth-Century Science*, Cambridge: Cambridge UP, 2008, p. 329:

The object of mathematics [in J.E. Montucla’s *Histoire des mathematiques*, 1799-1802] is the mutual relations of magnitude and number of any objects which are capable of increase or decrease. This explains the terminology ‘pure’ and ‘mixed’. Pure mathematics treats the relations between (variable or constant) quantities irrespective Of the objects they measure or count; mixed mathematics deals with quantities, and their relations, as they occur in natural objects which can be counted or measured. The terminology is indeed an appropriate one, better than the division into ‘pure’ and ‘applied’ now in use, which overlooks the dialectical nature of the use of mathematics and suggests that one either practises pure mathematics or takes a ready parcel of mathematics and applies it elsewhere.

WHAT IS NEW?

- The idea of explaining natural phenomena through mathematics is already present in Pappus.
  - What is new in Newton?
    - *Reduction* of «phenomena of nature to mathematical laws»
      - Relationship between geometry and mechanics
      - Mathematical principles in order to gain exactness
    - Introduction of «forces»: attempt at reducing all the phenomena to actions due to central forces (attraction and repulsion)
      - What about forces?

THE RELATIONSHIP BETWEEN GEOMETRY AND MECHANICS

THE RELATIONSHIP BETWEEN GEOMETRY AND MECHANICS (1)

We tend to think of mechanics as a sort of application of math: an applied mathematics…

So we have «pure» mathematics → mechanics

But Newton: «Since those who practice an art do not generally work with a high degree of exactness, the whole subject of mechanics is distinguished from geometry by the attribution of exactness to geometry and of anything less than exactness to mechanics. **Yet the errors do not come from the art but from those who practice the art**.»

So mechanics is as 'perfect' as geometry. Why? Because it is *mixed* mathemetics!

THE RELATIONSHIP BETWEEN GEOMETRY AND MECHANICS (2)

And note that for Newton the relationship goes in the opposite direction:

Mechanics → Geometry

Newton: «The description of straight lines and circles, which is the foundation of geometry, appertains to mechanics. Geometry does not teach how to describe these straight lines and circles, but postulates such a description. For geometry postulates that a beginner has learned to describe lines and circles exactly before he approaches the threshold of geometry, and then it teaches how problems are solved by these operations […]. **Therefore geometry is founded on mechanical practice and is nothing other than that part of universal mechanics which reduces the art of measuring to exact propositions and demonstrations**.»

THE RELATIONSHIP BETWEEN GEOMETRY AND MECHANICS (3)

Geometry and mechanics have equal degree of exactness (because of their mathematical method)

But they have a different scope:

- «Manual arts are applied to making bodies move […]; mechanics [is used] in reference to motion»
- «Geometry is commonly used in reference to magnitude»

→ So *rational mechanics* has not to do with machines but with motions, and it is the science of motion, «expressed in exact propositions and demonstrations»; that is, expressed in mathematical fashion.

THE INTRODUCTION OF «FORCES»

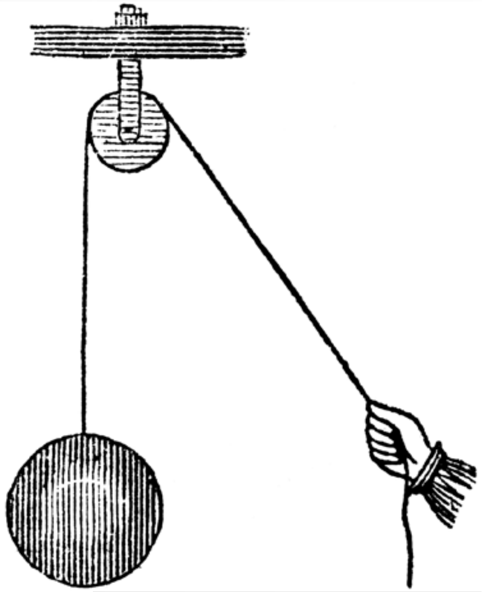
THE INTRODUCTION OF THE «FORCES» (1)

Even in this case, something inspiring is already present in the Ancients:

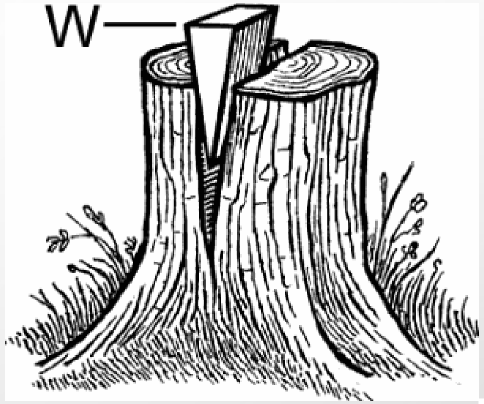
- «Manual arts are applied to making bodies move» and mechanics is used «in reference to motion».
- The ancients studied rational mechanics «in terms of the five powers that relate to the manual arts [i.e. Hero’s five mechanical powers: lever, pulley, wheel & axle, wedge, screw] and paid hardly any attention to gravity (since it is not a manual power) except in the moving of weights by these powers. But since we are concerned with natural philosophy rather than manual arts, and are writing about natural rather than manual powers, we concentrate on aspects of gravity, levity, elastic forces, resistance of fluids, and forces of this sort, whether attractive or impulsive.»

HERO’S FIVE MECHANICAL POWERS

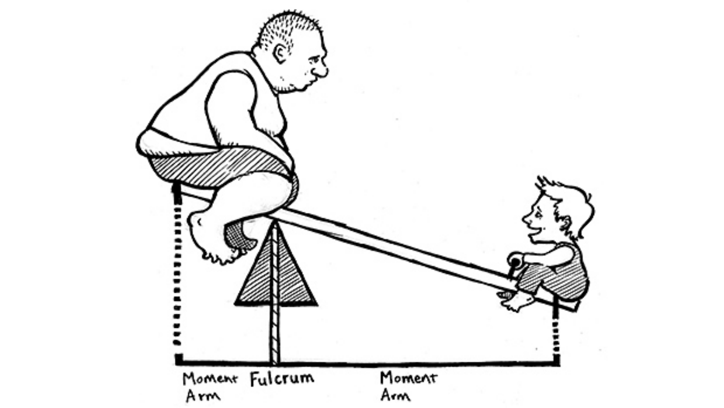
Pulley

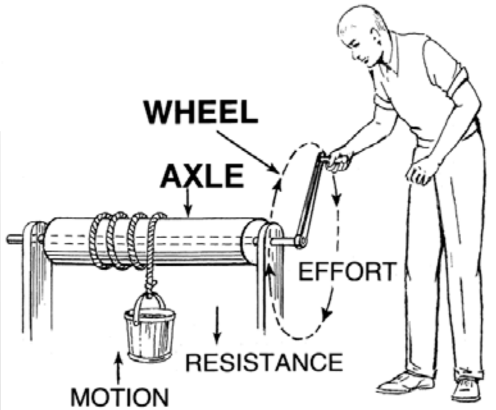


Wedge



Lever





Wheel & axle

Screw

THE INTRODUCTION OF THE «FORCES» (2)

Newton's «syllogism» about forces:

- Mechanics is useful in the investigation of nature;
  - So natural philosophy must deal with mechanics in order to comprehend natural phenomena;
- (Practical) Mechanics is a mathematized science of motions caused by «powers»;
  - Thus, «our present work sets forth **mathematical** principles of natural philosophy».
- If the analogy with (practical) mechanics holds, natural phenomena are motions caused by «powers».
  - Thus, «the basic problem of philosophy seems to be to discover the forces of nature from the phenomena of motions and then to demonstrate the other phenomena from these forces.»

THE INTRODUCTION OF THE «FORCES» (3)

- Newton's force in the *Principia*:
  - In the *Principia* there is practically one kind of forces: centripetal, «whether attractive or impulsive»:
    - They are discovered from the phenomena of motions [e.g. planets, comets, moon, sea-tides; gravity, levity, elastic forces, resistance of fluids…]
  - So in discovering forces, method is crucial:
    - Book 1 and Book 2 expose sets of «general propositions» (B1: mathematical methods & laws of centripetal forces; B2: theoretical and experimental study of the forces of resistance to motion in various types of fluids, particularly the fluid mechanics).
    - Book 3, «by means of propositions demonstrated mathematically in books 1 and 2, […] derive[s] from celestial phenomena the gravitational forces by which bodies tend toward the sun and toward the individual planets. Then the motions of the planets, the comets, the moon, and the sea are deduced from these forces by propositions that are also mathematical.»
    - Exactly here Newton expresses his dream of reduction: «If only we could derive the other phenomena of nature from mechanical principles by the same kind of reasoning!»

THE INTRODUCTION OF THE «FORCES» (4)

- Newton's force in the *Principia*:
  - Newton's dream of reduction: «If only we could derive the other phenomena of nature from mechanical principles by the same kind of reasoning!»
  - He goes further and introduces, as a sort of conjecture, repulsive forces – which are also directed towards *(from)* centers:
    - «Many things lead me to have a suspicion that all phenomena may depend on certain forces by which the particles of bodies, by causes not yet known, either are **impelled** toward one another and cohere in regular figures, or are **repelled** from one another and recede. Since these forces are unknown, philosophers have hitherto made trial of nature in vain. But I hope that the principles set down here will shed some light on either this mode of philosophizing or some truer one.»
      - *i.e.*: Natural philosophers did not succeded in correctly recognizing them, because they did not pose *mathematical* principles. But now, with such principles posed… Yes, we can!

THE INTRODUCTION OF THE «FORCES» (5.1): NEWTON OF THE *OPTICKS*

- Def. VIII of the *Principia* makes clear that «I use interchangeably and indiscriminately words signifying attraction, impulse, or any sort of propensity toward a center, considering these forces not from a physical but only from a mathematical point of view. Therefore, **let the reader beware of thinking that by words of this kind I am anywhere defining a species or mode of action or a physical cause or reason, or that I am attributing forces in a true and physical sense to centers** (which are mathematical points) if I happen to say that centers attract or that centers have forces.»
- But then, in the *Opticks*, you have Newton's speculations on the varieties of forces, «such as would be the Causes of Gravity, and of magnetick and electrick Attractions, and of Fermentations»
  - Newton calls them «**active principles**», and even «forces or actions»
  - Note that you have such speculations in the «Queries»

VIS PASSIVA

- Both in the *Principia* and in the *Opticks* there is an «inherent force» (*vis insita*) which is the force of inertia, a *passive* force that is (Def. III) «the power of resisting» every change of state of motion.
- Note that
  - Force of inertia is *inherent*, it is a property of (every kind of) matter, contrarily to the other forces, which are only mathematical assumptions.
  - Still, «passive» means that it is ‘activated’ by an external action: it is a typical relational ‘property’ that reveals itself only once an «impressed force» begins to act. If only one body would exist in the Universe, there would be no inertia at all!

THE INTRODUCTION OF «FORCES» (5.2): NEWTON OF THE *OPTICKS*

- So, *Principia* Def. VIII:
  - Forces are nothing but mathematical assumptions;
    - Whatever it means, they are not physical: **they are not properties of matter**
      - It’s not that matter is endowed with forces as something physical; they have more to do with mathematics: they depend on the mathematics involved in mechanics.
      - For example: something is moving *as if* some central force would be acting in a certain way; so you can calculate the shape of motion… Every judgement about its physical status is suspended. We don’t know anything about the ‘mechanism’, we only observe it.

- *Opticks*, Query 31:
  - There are some «active principles» – and without them the world would simply stop – **which are properties of the matter.**

FORCES OR NOT?

- Between these two «extreme positions» there is, to say at least, a tension, which caused trouble for the early Newtonians:
  - «Newton’s expositors had some tidying up to do: freeing the word ‘attraction’ from the rash interpretation […] and the ambiguous usage of Newton; answering the Cartesian insistence upon the epistemological superiority of ‘mechanical’ explanations; and clarifying the number and interrelations of the many short-distance attractions and repulsions mentioned by Newton from time to time. The tone for much of this work was set by ’sGravesande. [His] definition of gravitation returns to the most positivistic of Newton’s: gravity is not an occult quality but a manifest effect. ‘When we use the Words Gravity, Gravitation, or Attraction’, says Desaguliers, ‘we have a Regard not to the Cause, but to the Effect […]’, force meaning to Desaguliers and the Dutch Newtonians either momentum […] or the inertia that maintains motion (*vis insita*), not a physical cause acting between bodies […]. This defense was not made without cost, for it turned out that the quantities of fundamental interest to physicists were not forces as represented by Desaguliers and ’sGravesande, i.e. as macroscopic effects, but forces as microscopic causes, such as the suppositious mutual pull between all pairs of particles of matter.» (J. Heilbron, *Aether and Electricity in the 17th & 18th Centuries*, 1979)

GRAVITATIONAL LAW AND OTHER LAWS

- Of course Newton knows that these ‘microscopic circumstances’ also applied to the *Principia*:
    - he demonstrated (*Principia*, I, sect. 12-13) that his gravitational law, $F = \mathrm{G}\frac{m_1 m_2}{r^2}$, is strictly valid only in the spherical case (if masses are spheres). In the other cases it holds approximately.
    - So in the typical case of mechanics, when we are dealing with planets (quasi-spheres) and large distances, we can abstract and say: well, gravitation and other forces, which are inverse-square forces, are just effects: they only stand for velocity changes (change in acceleration or in momentum)
- But in doing microphysics we can’t, since we deal with medium and short distances: we can’t abstract from the composition of matter (Newton supposed the spheres to be homogeneous), we can’t abstract from the form of the bodies involved (magnetic rods, for example), we can’t abstract from the medium (how to explain that gases diffuse if not through a repulsive virtue of air? → Stephen Hales’s *Vegetable Staticks*)
    - In all these cases the early Newtonians simply *measured* properties of matter – not mathematical assumptions! Not all the bodies are electrical, not all the bodies are magnetical, chemical effects only follow from particular states of matter and air…
    - On the contrary, in Newton’s gravitational law the only crucial thing is **distance**!
- In a certain sense, which should be precised, we could say that:
    - In mechanics *force* is a concept derived by motions
    - In other «microphysical» fields, *force* is a concept derived by measure

NEWTON'S GRAVITATIONAL LAW AS A LAW ON MOTIONS

- We don't know what gravity is; but we know that

  «*Experimental Proposition*. Bodies set opposite each other induce in each other, under certain circumstances to be specified by experimental physics, contrary *accelerations* in the direction of their line of junction. (The principle of inertia is included in this)»

  (E. Mach, *The Science of Mechanics*, p. 243.)

- Note that:
  - No difference with the law of gravitational *force* – as Desaguliers claimed, we should have regard for the effects, and the effects (what we observe) are properly velocity changes.
    - Mach describes this sentence as an «experimental proposition»!
  - The principle of inertia is included since the «exp. proposition» indirectly says that if other bodies are absent, the one body we observe is not accelerated.

WHAT ARE FORCES?

- So forces are, in Mach's view, nothing but 'economic symbols', a by-product of our more or less efficient mathematical notation:

  «*Definition*. Moving force is the product of the mass value into the acceleration induced in that body»

  (E. Mach, *The Science of Mechanics*, p. 243.)

- Note that this last sentence in symbols is what we are used to call «the second law of Dynamics» or briefly «Newton's second law»:

$$\boldsymbol{F} = m\boldsymbol{a}$$

  which *is* and in the same time *is not* Newton's!

NEWTON'S SECOND LAW

- «A change in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed.»

$$\Delta v \propto F_i$$

  where F_i stands for «force impressed».

- In other words, Newton's law seems to be more explicable in terms of the (Cartesian) notion of momentum or quantity of motion. And…

$$\boldsymbol{F} = m\Delta\boldsymbol{v} \quad \neq \quad \boldsymbol{F} = m\boldsymbol{a}$$

BUT…

FROM NEWTON TO EULER

- Who envisaged the form «$F=ma$» was definitely Euler, not Newton
  - As shown by Truesdell (1968), only by insight we may say that «$F=ma$» is Newton – but Euler was convinced that this (expressed in yet another form) was in fact a «new principle of mechanics»)
- So are we definitely incorrect? What is really new in Euler that is not present in Newton?
  - A clear(er) insight of the Calculus (which allows you to speak in terms of a continuously acting force, i.e. of an acceleration)
  - The extended context of the *mathematical general* science of mechanics (all motions are to be accounted for, bodies subject to *any* forces, in *any* state and so on)