The background of the slide is a light gray gradient with several realistic water droplets of various sizes scattered across it. The droplets have highlights and shadows, giving them a three-dimensional appearance. They are located in the top-left, top-center, and bottom-right areas of the slide.

SESSION VII

A PARADOX AND ITS POSSIBLE SOLUTION
IN THE NEWTONIAN FRAMEWORK

OUTLINE

- Gravity being only attractive, Newton's mechanics let room for the so-called «cosmological paradox» (which we do not examine in all its breadth, but only as Bentley's questions and Newton's reaction are concerned)
 - The key idea is that all masses which form the Universe convene in a shapeless, unmoving bulk
 - We take into consideration some problems related to it, essentially the theological problem, which is the root of the paradox
- We consider a possible answer within the Newtonian physics
- In this context we introduce Boscovich's theory and observe that it is a theory «on motions» more than «on forces» (despite the title of his main work, *Theoria philosophiae naturalis redacta ad unicam lege virium in natura existentium*)

NEWTON'S CAREER AND CORRESPONDENCE

«Newton's philosophical training and work began early in his intellectual career while he was an undergraduate at the Trinity College in Cambridge in the early 1660s. The notebooks that survive from that period indicate his wide-ranging interests in topics philosophical, along with a reasonably serious acquaintance with the great "moderns" of the day, including Robert Boyle, Hobbes, Gassendi, and especially Descartes. Later in his life, Newton corresponded directly with a number of significant figures in natural philosophy, including Boyle, Huygens, and Leibniz, and he developed personal relations with many others, including Henry More and John Locke. Newton's primary works, of course, are *Philosophiæ naturalis principia mathematica* and the *Opticks*. Each went through three successive editions during Newton's lifetime, which he oversaw under the editorship of various colleagues, especially **Richard Bentley**, Samuel Clarke, and Roger Cotes.»

(A. Janiak, Introduction to Newton, *Philosophical Writings*, Cambridge UP, p. xiii)

Best biography of Newton: Richard S. Westfall, *Never at Rest*, Cambridge UP, 1981.

THE NEWTON–BENTLEY RELATIONSHIP

- At the time of their correspondence (1692–1693), Richard Bentley – a classical scholar and **theologian** of Cambridge (whom he never became a fellow) – was prebendary (“canonico”) and chaplain to the Bishop of Worcester.
- During 1692 he delivered a course of sermons under the title of *A confutation of Atheism*, published in 1693. It is in this connection that he asked for Newton’s advice, whose ideas he wanted to exploit in order to defend his own theses.
- Their correspondence include at the present 4 letters by Newton and 1 letter by Bentley.

THE NEWTON–BENTLEY CORRESPONDENCE: BENTLEY’S QUESTION ABOUT THE UNIVERSE FATE

Bentley’s question

- How did the universe develop according the theory of universal gravitation?
 - Under the assumption of the initial condition: uniform distribution of matter

Newton’s answers (1692–12–10)

- Two cases:
 - Finite space: all matter tends to «fall down into the middle of the whole space & there compose one great spherical mass»
 - Infinite space: some matter «would convene into one mass & some other into another so as to make an infinite number of great masses, scattered at great distances from one to another throughout all ye infinite space.» So all remains in equilibrium.

Note: Newton will remember Bentley’s question in the *Opticks*: Query 28.

BENTLEY'S «QUERIES» IN THE MIRROR OF THE QUERY 28 OF THE *OPTICKS*

Q 28. [...] For rejecting such a medium, we have the authority of those the oldest and most celebrated philosophers of *Greece* and *Phoenicia*, who made a *vacuum*, and atoms, and the gravity of atoms, the first principles of their philosophy; tacitly attributing gravity to some other cause than dense matter. Later philosophers banish the consideration of such a cause out of natural philosophy, feigning hypotheses for explaining all things mechanically, and referring other causes to metaphysics: whereas the main business of natural philosophy is to argue from phenomena without feigning hypotheses, and to deduce causes from effects, till we come to the very first cause, which certainly is not mechanical [against Descartes]; and not only to unfold the mechanism of the world, but chiefly to resolve these and such like questions.

What is there in places almost empty of matter, and **whence is it that the sun and planets gravitate towards one another**, without dense matter between them? Whence is it that nature doth nothing in vain; and whence arises all that order and beauty which we see in the world? To what end are comets, and **whence is it that planets move all one and the same way in orbits concentric, while comets move all manner of ways in orbits very eccentric** [Bentley's second question]; and **what hinders the fixed stars from falling upon one another?** [Bentley's first question: stability paradox]

BENTLEY'S OBJECTION

An objection

- Under Newton's assumptions each particle would be in equilibrium for ever and no motion would occur, for every particle would be pulled in every direction by equal and contrary forces (similar to a force of cohesion).

Newton's answers (1693–01–17)

- The condition of an equal pull in every direction, which would result in a «mechanical death» – or perhaps a mechanical abortion – is very improbable, since it implies that all the particles are exactly placed at the beginning.

THE PARADOX AND ITS THEOLOGICAL SOLUTION

- As Newton recognizes, Bentley's condition, as improbable as it might be, is not impossible «at least for a divine power».
 - If God is good (for humans), all right! But God's decisions are good because are God's decisions, so what if God chooses *for his own «good» reasons* an inconvenient setup for humans?
- In other words, Newton's system requires at least a «provident God» who acts or does not act for Universe's safety – for saving the Universe from a deadly equilibrium..., that is for *preserving*
- ...and of course Newton needs Good for *initiating* the Universe too (Query 28 of the *Opticks*)
- But Bentley went further:
 - Imagine that matter is infinite and uniformly distributed in an infinite space
 - So you have infinite gravitational pull in any direction, and all is at rest
 - If God adds a finite force to a single particle, he is adding a finite quantity to an infinite quantity, and $x + \infty = \infty$
 - Yeah, Newton says, but remember that all the particles (say the stars) are infinite in number and «evenly distributed». Thus, they can be paired so that their actions cancel out one another
 - The only force that remains is that added, as small as it is.

NEWTON VS NEWTON

NEWTON'S UNSAID ASSUMPTIONS

- The problem is that Newton's solution is not granted: we cannot be sure that stars' gravitational pull cancels out on the whole, unless the stars are accurately placed at the beginning (which was, in fact, Newton's answer to Bentley's objection!)
 - Following Newton, the gravitational pull of star pairs cancels out only *approximately*, but this again implies Newton's argument against Bentley: there remains a force that compels every particle to «convene» somewhere, until a mechanical implosion of the entire universe takes place
- More or less, Newton is assuming two things in order to avoid both the mechanical abortion and the mechanical catastrophe (implosion) of the universe:
 - *Perfect* uniform distribution of the bodies in the universe, particularly the stars (bodies with great gravitational force)
 - Stars are very far one from the other: so the gravitational force is very small at big distances, and at infinite distance it is infinitely small
 - But again: very small or infinitely small is not zero! So in very large time all should «convene» and the mechanical catastrophe occurs – only God could prevent it...

A NECESSARY GOD

- Newton needs God for two businesses:
 1. For initiating the universe with certain very peculiar circumstances
 2. For preserving the stability of the universe, i.e. that condition of *near-equilibrium* required for Newton's system itself
 3. (For granting the absoluteness of space)
- Ok for business 1, maybe not with enthusiasm, but we take it for granted
- But 2 is very demanding! One might think that a God who makes the universe as imperfect as to require his continuous intervention is an imperfect God!
 - If you are ready to consider the last interpretation, then Bentley's paradox on the instability of Newtonian system signals a real deficiency in Newtonian physics.

BOSCOVICH'S SOLUTION

- In 1758/1763 Ruggiero (Rudjer) Boscovich, a Dalmatian-born Jesuit, tried to give an answer by envisioning a change in Newton's law of gravitation.
- What follows is Boscovich's account of the Bentley-Newton's aporia (*Theoria*, nn. 404-405) about 70 years later:

404. It is usually objected to the universal gravitation of Newton, that in accordance with it the fixed stars should by their mutual attraction approach one another, & in time all cohere into one mass [Bentley's criticism]. Others reply to this that the universe is indefinitely extended, & therefore that any one fixed star is equally drawn in all directions [Newton's reply]. [...] Others fall back on the immense distance, which they say will not permit the motion arising in the fixed stars from the force of gravity to be perceived by the senses, even after an immense number of ages [Newton's implicit assumption].

[Boscovich's argument, concession] In this they assert nothing but the truth: suppose we consider the fixed stars equal & similar to our sun, or at any rate the amounts of light that they emit, as not being far different from the ratio of their masses. In this case, since also the force is proportional to the masses, & in addition both force & light decrease in the inverse ratio of the squares of the distances, it must be that the force of gravity of our solar system on all the stars is to the force of our gravity on the sun. And the latter is many times less than the force of gravity of our heavy bodies on the earth, as the total light which comes from all the stars is to the light which comes from the sun & this ratio is the same as the ratio of night to day in respect of light [connection Olbers's/Bentley's paradox]. How slight is the motion that can arise from this in the time (the comparatively short time available for observation) nobody can fail to see. Even if all the fixed stars were situated in the same direction, the motion could be considered as absolutely nothing.

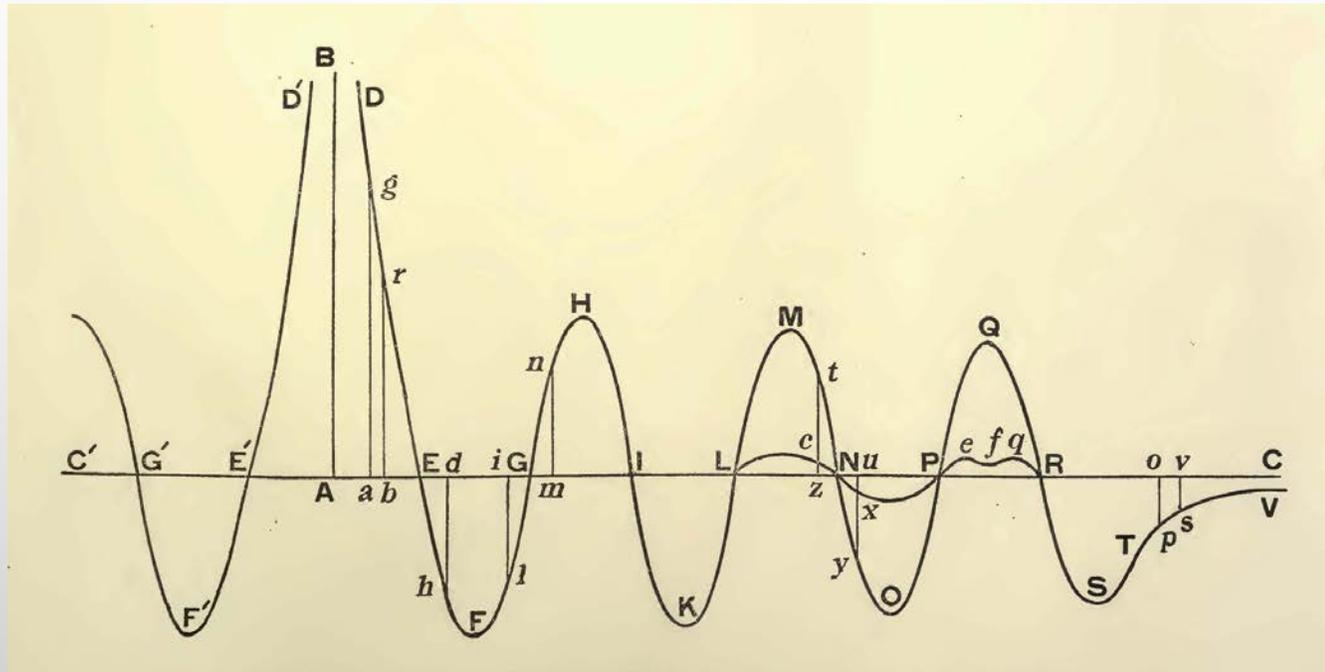
HOWEVER...

405. However, our period of life & memory, in comparison with the immense number of ages perchance to follow, is almost as nothing. Therefore, if universal gravitation extends indefinitely with the same law [...], not only our solar system, but the universe of corporeal nature, would, little by little in truth, but still continuously, recede from the state in which it was established, & the universe would necessarily fall to destruction. All matter would in time be conglomerated into one shapeless mass, since the gravity of the fixed stars on one another will not be cancelled by any oblique or curvilinear motion. That this is not the case cannot be absolutely proved [**for God does what he will, and whatever he does is good**]; & yet a Theory which opens up a possible way to avoid this universal ruin, in the way that my Theory does, would seem to be more in agreement with the idea of Divine Providence [**Remember, Boscovich was a Jesuit: more or less, Good and humans agree on the meaning of good and evil!**]. **For it may be that [...] the last arc of my curve, which represents gravity, after it has reached distances greater than the greatest distances from the Sun of all the comets that belong to our solar system, will depart very considerably from the hyperbola having its ordinates the reciprocals of the squares of the distances, & once more will cut the axis & be twined about it.**

BOSCOVICH'S LAW OF FORCES

Above the axis:
repulsion

Under the axis:
attraction



Abcissas:
distances

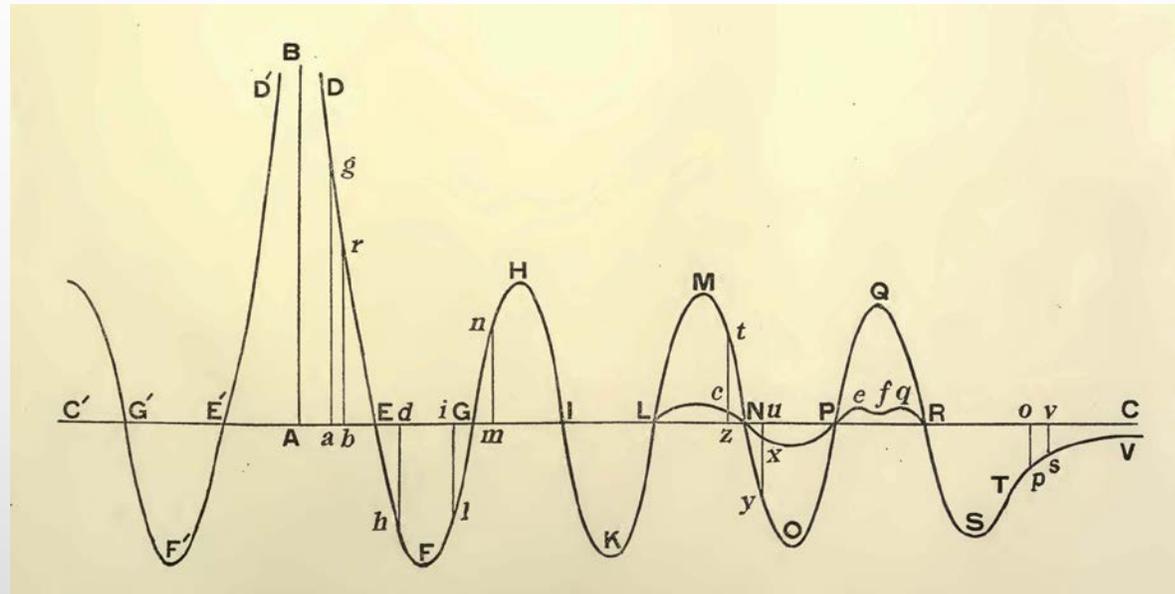
Ordinates:
intensity of the
force

Boscovich's curve of forces — more precisely, of only one force acting at a distance, which is alternatively attractive and repulsive (*only* depending on distance): it grows infinitely repulsive when the distance is infinitely small and approximates Newton's inverse square law when distances are very big, say on the scale of the Solar System.

BOSCOVICH'S LAW OF *FORCES*?

Above the axis:
repulsion

Under the axis:
attraction



Abscissas:
distances

Ordinates:
intensity of the
forces

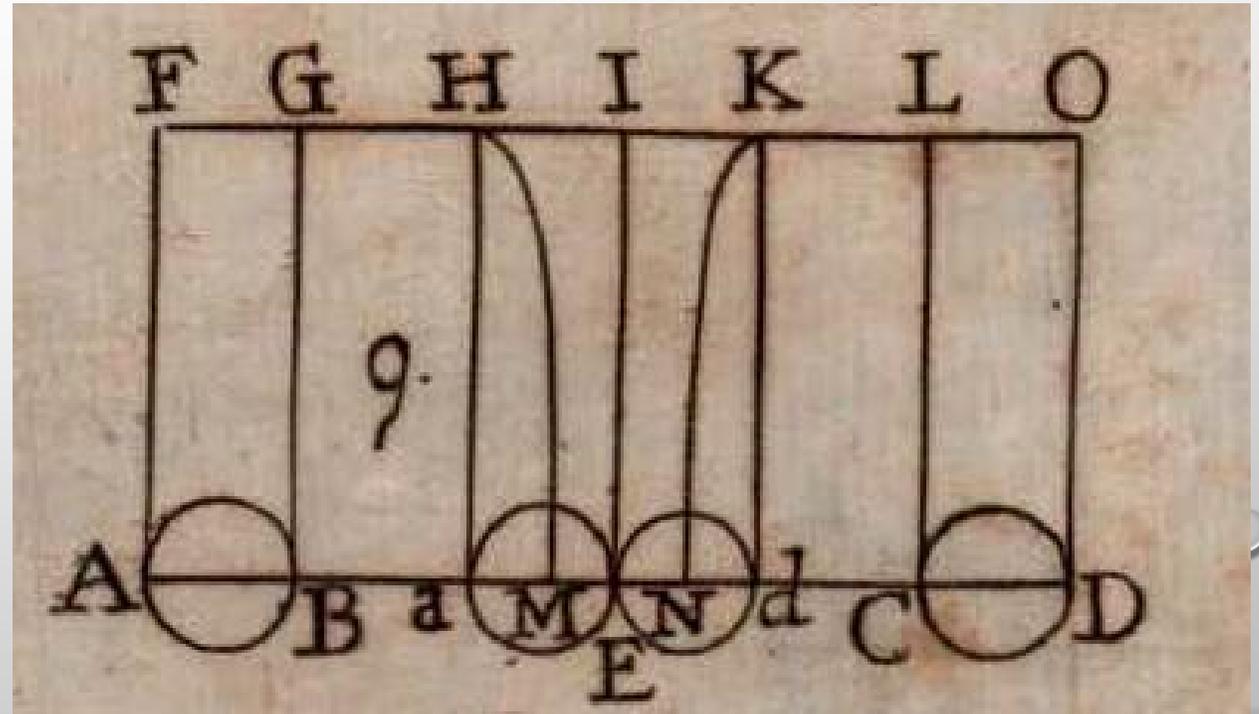
Imagine to be a dimensionless, material point. You may start *very near* point A, but not exactly in A (because of asymptotic repulsion); you're under a repulsion condition till you reach E, which is a "limit point" (equilibrium). If you go further, you encounter some difficulties, because now you are attracted (meaning that E represents a strong cohesion...). Now, the properties of forces only depend on the geometrical (analytical) properties of the curve. Properly there are no forces at all, you see only motions here – and the fact that every *distance* is endowed with a determination of receding or approaching depending on distances. The only thing the law dictates is that if you are at a determinate distance you assume a determinate disposition. This is what you call a force! Quasi a conception of potential...

BOSCOVICH AND COLLISION

- Boscovich introduced his curve in the context of the theory of bodies collision (1745, *De viribus vivis*)
- Two explications:
 - Cartesians (according to Descartes's rules of collision): a sudden change in velocity is implied. If two equal bodies collide with equal speeds, then they immediately stop in the collision and speed becomes suddenly 0.
 - This was challenged by Leibniz, e.g. in the *Lettre sur un principe general utile à l'explication des loix de la nature* (1687), and this general principle is what he later named the law of continuity. For the Leibnizians, you can avoid jumps in velocity by thinking of bodies as soft, flexible entities. So in collision, immediately after contact their parts modify their forms progressively, that is their particles slow down progressively *after* contact. Boscovich agrees that jumps are to be avoided because they break the continuity, which is a general principle of order in Leibnizian terms. But he disagrees with the argument. Anyway... this is his idea

BOSCOVICH'S EXPLANATION

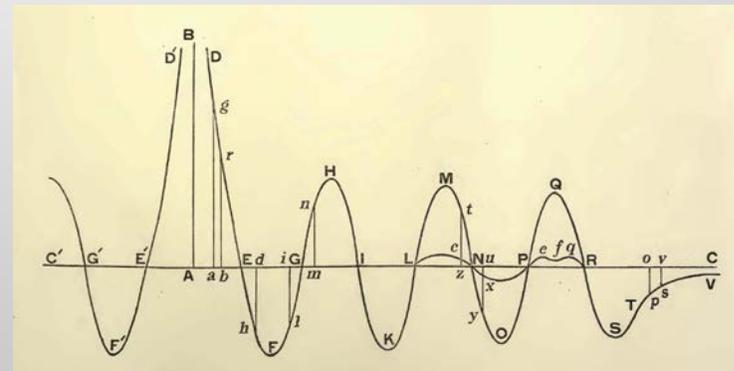
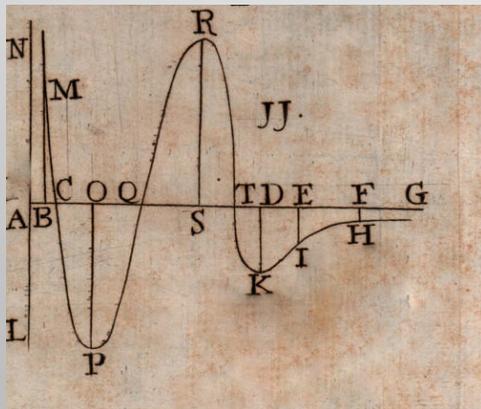
Let's take two equal spheres AB and CD moving with equal velocities, which we represent as the lines AF, GB, aH for the sphere AB; and DO, CL, dK for the sphere CD. (Velocities are intended for every particles or every single point forming each sphere, so the velocity OD is e.g. the velocity of the point D of the sphere CD.) For Boscovich, the problem is that in the instant of collision, all velocities diminish gradually (like HM, KN: each part presses upon each other, is a little bit displaced and ends up with velocity = 0), except the velocities of the surfaces at the points B and C. In fact, the velocities should be given by lines equal to each other (like FA, OD...) until the positions *a* and *d*. Then, when the spheres have reached the positions *a* and *d*, the surfaces (the first particles) at the points B and C reach the point of contact E with their velocities EI — which suddenly disappear, because the particles have no time nor space to change the velocity progressively. So Boscovich's question still remains: how is possible to avoid the jump?



BOSCOVICH'S ANSWER

According to Boscovich, a very general principle of continuity holds, so that:

- 1) velocity can be nor instantaneously generated neither destroyed;
- 2) therefore, bodies must slow down when approaching one another — to account for this, a repulsive force must be acting at smallest distances... so that as bodies move nearer, and come very near, attraction decreases and the force ends up in a repulsion which slows them down and finally pushes them back until a point of equilibrium, where repulsion is balanced out by attraction (let's say in C, that is a zero on the x-axis). The obvious consequence is that contact doesn't exist. So, that's Boscovich's curve!



GEOMETRICAL APPROACH

- What is crucial is the geometrical approach:
 - Mechanical systems are only discussed in geometrical terms
 - Leibnizian theory of collision is rejected on a «geometrical» basis
 - Forces at a distance only depends on the geometry («analytical properties») of the curve
 - No physical assumptions about forces!!!