Identity and Trinity*

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All theologians understand that the central problem involving the mystery of the Trinity is to explain the possibility that there is one God but three persons in God without falling into contradiction. Many, if not most, contemporary theologians who write about the Trinity believe that the key to the solution of this problem lies in adequately analyzing one or more of the concepts of a person, a nature, a substance, or God, or in constructing some new concept, say, that of "persons in community" or "a society of persons."1 Such theologians very often talk about the metaphysics or ontological dimensions of these concepts and explain the mechanisms, the ontological mechanisms, operating in the Trinity. The almost clinical accounts of the love life and intimate activities of the Trinity are at best very hard to reconcile with the spirit of the Trinity as a mystery.

Worse, such discussions mislocate the source of the problem, which is that Father, Son, and Holy Spirit are somehow identical and yet not identical. As the notion of identity is generally construed, this is incoherent no matter how "person," "nature," or what-have-you is analyzed. In this paper I want to sketch an account of identity that will forestall the seeming incoherence of the Trinitarian doctrine.

I

In On Christian Doctrine I,5,5 St. Augustine provides a clear statement of the orthodox doctrine of the Trinity.2

Thus there are the Father, the Son, and the Holy Spirit, and each is God, and at the same time

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2 For the purposes of this paper, "orthodoxy" refers to the doctrines formulated in the Nicene Creed, the Nicene-Constantinople Creed, the Creed of Epiphanius, and the Athanasian (Quicumque vult) Creed.

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time all are one God; and each of them is a full substance, and at the same time all are one substance. The Father is neither the Son nor the Holy Spirit; the Son is neither the Father nor the Holy Spirit; the Holy Spirit is neither the Father nor the Son. But the Father is the Father uniquely; the Son is the Son uniquely; and the Holy Spirit is the Holy Spirit uniquely.

The central theses of this passage include the following:

1. There is only one God.
2. The Father is God.
3. The Son is God.
4. The Father is not the Son.
5. The Holy Spirit is God.
6. The Holy Spirit is not the Father.
7. The Holy Spirit is not the Son.

The doctrine of the Trinity as captured in these seven propositions is paradoxical, if for no other reason than that they seem to form an inconsistent set, and accordingly not all of them can be true; one or more of them must be false. If 1–7 do form an inconsistent set, then it follows that part of orthodoxy is false.

Propositions 1–7 seem to be inconsistent several times over, and one of these inconsistencies can be shown to follow from 1–4. Thus I shall restrict my discussion to these four propositions. In fact, it is usual for Trinitarian discussions to focus on the Father and the Son; if they can be logically reconciled, then the "love" thereby generated can be extended to include the Holy Spirit.

The apparent inconsistency involved in propositions 1–4 can be shown by deriving a contradiction from them. To do this, we have to show what steps in reasoning justify the inferences that lead to the contradiction. What we are committed to is a proof of the contradiction. We will of course require that the proof be rigorous; for if it were not, we would not be forced to accept the conclusion. Since the best, if not the only, place to find the kind of rigor demanded is in the discipline of formal logic, it is to formal logic that we must go for proof.

All the resources we need for the required proof can be found in any standard, first-order predicate logic with identity theory. The addition of identity theory to the predicate logic consists in selecting some predicate to express identity and adding rules that are suitable and sufficient to permit inferences justified by the concept of identity. For our present purposes we need only to add rules that are justified by the fact that identity is an equivalence relation, that is, a relation that has the three characteristics of reflexivity, symmetry, and transitivity. Since identity is reflexive, the following principle holds:

Everything is identical with itself.

In symbols,

\[(x)x = x.\]
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Under the rubric of Laws of Thought, many philosophers have knighted this principle the “Law of Self-Identity” and have made extravagant claims for its importance in metaphysics. We shall not need to formulate a rule of inference for reflexivity. Next, since identity is symmetrical, the following principle holds:

If \( x \) is identical with \( y \), then \( y \) is identical with \( x \).

In symbols,

\[(x)(y)(x = y \rightarrow y = x).\]

The content of the principle of symmetry can be captured in a rule of inference which allows one to reverse the singular terms flanking the identity sign:

*The Rule of Symmetry.*—Where \( \alpha \) and \( \beta \) are individual constants, if something of the form \( \neg \alpha = \beta \) is a premise or a line of proof, enter \( \neg \beta = \alpha \) as a line of proof.

Finally, since identity is transitive, the following principle holds:

If \( x \) is identical with \( y \) and \( y \) is identical with \( z \), then \( x \) is identical with \( z \).

In symbols,

\[(x)(y)(z)[(x = y \& y = z) \rightarrow x = z].\]

The content of the principle of transitivity can be captured in the following rule of inference:

*The Rule of Transitivity.*—Where \( \alpha \), \( \beta \), and \( \gamma \) are individual constants, if \( \neg \alpha = \beta \) and \( \neg \beta = \gamma \) are premises or lines of proof, then enter \( \neg \alpha = \gamma \) as a line of proof.

We are now prepared to derive a contradiction from 1–4 using the rules of natural deduction, symmetry, and transitivity.

1. \((\exists x)[Gx \& (y)(Gy \rightarrow y = x)]\)
2. \(Gf\)
3. \(Gs\)
4. \(f \neq s\)
   \(\alpha. \quad [Ga \& (y)(Gy \rightarrow y = a)]\) \(1, \) existential instantiation.
   \(\beta. \quad (y)(Gy \rightarrow y = a)\) \(\alpha, \) simplification.
   \(\gamma. \quad (Gf \rightarrow f = a)\) \(\beta, \) universal instantiation.
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[\[ \delta. \quad f = a \quad \ \text{2, } \gamma, \text{modus ponens.} \]
[\[ \epsilon. \quad (Gs \to s = a) \quad \beta, \text{universal instantiation.} \]
[\[ \zeta. \quad s = a \quad \ \text{3, } \epsilon, \text{modus ponens.} \]
[\[ \eta. \quad a = s \quad \ \zeta, \text{symmetry.} \]
[\[ \theta. \quad f = s \quad \delta, \eta, \text{transitivity.} \]
[\[ \iota. \quad (f = s \& f \neq s) \quad 4, \theta, \text{conjunction.} \]

II

One way to resolve the seeming inconsistency is to deny one or more of propositions 1–4. Although any one will do, the most likely is 3 or 4. Yet the denial of either is heresy; each is part of the deposit of faith. Reason and revelation seem to have engaged forces in civil war. Reason cannot accept revelation’s seeming contradiction, while revelation cannot allow reason to abandon any article of faith. In deciding which side to support, one must consider that, if faced with the alternatives of being a heretic and asserting a contradiction, the rational person will always choose heresy and trust himself to the mercy of God. A rational man can put no faith in contradiction.

Arius and Sabellius were rational men who could see no way to reconcile propositions 1–4. Their solutions to the problem, though similar in spirit, were different in letter. Arius denied 3; Sabellius denied 4. Each man can be seen as building his doctrine upon an argument.

Arius’s Argument
1. There is only one God.
2. The Father is God.
4. The Father is not the Son.
3'. Therefore the Son is not God.

Sabellius’s Argument
1. There is only one God.
2. The Father is God.
3. The Son is God.
4'. Therefore the Father is the Son.

There is a formal proof of the validity of each of these arguments in first-order logic. Sabellius’s argument can be proved with the rules already provided.

Sabellius’s Argument
1. (\(\exists x)(Gx \& (y)(Gy \to y = x))\)
2. \(Gf\)
3. \(Gs\)
\[ \alpha. \quad (Ga \& (y)(Gy \to y = a)] 1, \text{existential instantiation.} \]
\[ \beta. \quad (y)(Gy \to y = a) \quad \alpha, \text{simplification.} \]
\[ \gamma. \quad (Gf \to f = a) \quad \beta, \text{universal instantiation.} \]
\[ \delta. \quad f = a \quad 2, \gamma, \text{modus ponens.} \]
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\( (Gs \rightarrow s = a) \quad \beta, \text{universal instantiation.} \)

\( s = a \quad \gamma, \text{modus ponens.} \)

\( a = s \quad \zeta, \text{symmetry.} \)

\( f = s \quad \delta, \eta, \text{transitivity.} \)

Arius’s argument can be proved with the addition of another rule of inference, justified by the concept of absolute identity. The additional rule of inference follows from the same principle that entails identity as an equivalence relation. This principle is sometimes called Leibniz’s Law:

\[
\text{x is identical with y just in case whatever property Z that x has y has also.}
\]

In symbols,

\[
(x)(y)[x = y \leftrightarrow (Z)(Zx \rightarrow Zy)].
\]

As indicated by the biconditional connective “just in case,” this statement splits into two corollaries. One of these has traditionally been called the Identity of Indiscernibles:

\[
\text{If whatever property Z that x has y has also, then x is identical with y.}
\]

In symbols,

\[
(x)(y)[(Z)(Zx \rightarrow Zy) \rightarrow x = y].
\]

The other corollary, often mistakenly confused with the first, is properly termed the Indiscernibility of Identicals:

\[
\text{If x and y are identical, then whatever property Z that x has y has also.}
\]

In symbols,

\[
(x)(y)[x = y \rightarrow (Z)(Zx \rightarrow Zy)].
\]

The principle of Indiscernibility of Identicals justifies the additional rule of inference that is needed to prove the Arian argument:

\textit{The Rule of Substitutivity of Identicals.}—Where \( \alpha \) and \( \beta \) are individual constants, if
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"... α --- " and "α = β" occur as a premise or line of proof, then enter "... β --- " as a line of proof.

We are now prepared to provide a proof for the Arian argument, translated into an obvious symbolism.

Arius's Argument
1. (∃x)[Gx & (y)(Gy → y = x)]
2. Gf
4. f ≠ s
α. [Ga & (y)(Gy → y = a)] 1, existential instantiation.
β. (y)(Gy → y = a) α, simplification.
δ. f = a
ε. (Gs → s = a) 2, γ, modus ponens.
η. s ≠ a 4, δ, substitutivity.
ζ. -Gs 5, symmetry.

At this point there may seem to be no rational way to eliminate the inconsistency without falling into heresy. Thus harried by inconsistency and beset by heresy, one might be tempted to irrationality and claim, as Peter Damian did, that God's power is not limited by anything, not even the bounds of logical possibility. Thus Peter believed that God could restore a fallen woman to her virginal state.3 If one can believe that, one can believe that God is one and three. Or one might be tempted to give as a reason for belief, as Tertullian did for Christ's resurrection, the fact that it was impossible.4

Lead us not into temptation. Religion does not excuse irrationality. Theologians are subject to the same canons of reason as mathematicians. Is not God Wisdom itself,"is not the Son the Logos? There is neither sense nor salvation in incoherence, and Kierkegaard in his death-defying leap of faith into the abyss of absurdity has no good hope that ye will be caught by any angels.

This notwithstanding, to be faced with a choice between being a heretic and asserting a contradictory proposition is not to find oneself in the best possible world; it is better to be orthodox and rational, and I want to show that these qualities are compatible, at least in the present case.

III

We have three problems to solve. The first is the inconsistency of 1–4, the second is the heretical argument of Arius, and the third is the heretical argument of Sabellius. I shall take these problems in order.

3 De divina omnipotentia, 4.
4 De carne Christi, 5.
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There is a clue to the solution of the first problem in other types of sentences that seem to be contradictory. Despite appearances, sentences such as “Dillinger is good” and “Dillinger is not good” need not be taken as incompatible; each can be used to express a proposition that is consistent with the other. The first sentence might be used to express the proposition that Dillinger is a good killer, the second to express the proposition that Dillinger is not a good person. In such cases, the speaker depends upon the context of the speech situation to make clear to the hearer what has to be added to his uttered sentence in order to make what he says complete. In other words, the predicate “is good” is incomplete. A thing is good in one respect or another; nothing, not even God, is good absolutely. God is good as a father but not good as a liar; he is good as a king but not good as a tyrant.

The identity predicates “is,” “is the same as,” and “is identical with” are incomplete in the same way. Nothing is identical with something absolutely; but only in a certain respect. Nothing is merely self-identical; there is no bare self-identity. Everything is the same such-and-such or so-and-so as something, and if it is not a such-and-such or so-and-so, it fails to be the same such-and-such or so-and-so. I am the same man as myself, but I am not the same house as myself. What Aristotle and some of his followers call numerical identity is a fiction. In short, identity is not absolute but relative. 5

Since 4 and 4' do not specify the respect in which the Father and the Son are supposed to be the same and not the same, they are incomplete and thus do not necessarily contradict each other. To determine whether they do contradict each other, they must be completed to include the respect of sameness. The respects alluded to here are well known; 4 and 4' give way to

8. The Father is not the same person as the Son.
8'. The Father is the same God as the Son.

Prima facie, the contradiction has been resolved: 8 and 8' are not formally contradictory. If only it were this easy; it may be that we are being misled by the grammatical form of 8 and 8'. To determine whether the contradiction has in fact been resolved, we must establish their logical form.

The standard way of putting 8 and 8' is

8. \( (P_f & P_s & f \neq s) \),
8'. \( (G_f' & G_s & f = s) \).

These obviously will not do. For $8$ and $8'$ are inconsistent; the former entails

4. $f \neq s$

(by simplification), while the latter entails

4'. $f = s$

(by the same rule), and this puts us back where we started.

The answer to this difficulty is to reject the proffered construal of the form of $8$ and $8'$; it represents identity as being absolute and ignores the all-important claim that identity is relative. Since we are claiming that identity is relative, an expression of the form $\neg \alpha = \beta^\phi$ is incomplete; it must be supplemented with a specification of the respect in which $\alpha$ is identical with $\beta$; that is, if something of the form $\neg \alpha = \beta^\phi$ is true, then $\alpha$ is the same something as $\beta$. Accordingly we shall use

$$\neg \alpha = \beta^\phi$$

\[\phi\]

to symbolize that $\alpha$ is the same $\phi$ as $\beta$. In particular, we shall use $G$ to symbolize “is the same God as” and $P$ to symbolize “is the same person as.” Let us then symbolize $8$ as

8. $f \neq s$

\[P\]

and $8'$ as

8'. $f = s$.

\[G\]

Propositions $8$ and $8'$ are formally consistent with each other. Further, $1$–3, $8$, and $8'$ form a consistent set. Relativizing the identity predicate in 1 and adding the other premises, we can see that the analogue of the original proof of inconsistency is now harmless. We prove not

\[1. \ (f = s \ & f \neq s)\]

but only

\[\kappa. \ (f = s \ & f \neq s)\]

\[G \ P\]

which is formally consistent:

1. $(\exists x)[Gx \ & (y)(Gy \rightarrow y = x)]$

2. $Gf$

3. $Gs$

4. $f \neq s$

\[P\]

\[\alpha. \ (Ga \ & (y))Gy \rightarrow y = a)]\]

1, existential instantiation.

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\[
\begin{align*}
\beta. & \quad (y)(Gy \rightarrow y = a) \\
\gamma. & \quad (Gf \rightarrow f = a) \\
\delta. & \quad f = a \\
\epsilon. & \quad (Gs \rightarrow s = a) \\
\zeta. & \quad s = a \\
\eta. & \quad a \equiv s \\
\theta. & \quad f = s \\
\kappa. & \quad (f = s & \land f \neq s) \\
\end{align*}
\]

\[\alpha, \text{ simplification.} \]
\[\beta, \text{ universal instantiation.} \]
\[2, \gamma, \text{ modus ponens.} \]
\[\beta, \text{ universal instantiation.} \]
\[3, \epsilon, \text{ modus ponens.} \]
\[\zeta, \text{ symmetry.} \]
\[\delta, \eta, \text{ transitivity.} \]
\[4, \theta, \text{ conjunction.} \]

Notice that we used symmetry and transitivity in this proof. All the relative identity predicates are equivalence relations. If the Father is the same God as the Son, then the Son is the same God as the Father. If the Alpha is the same person as the Son and the Son is the same person as the Omega, then the Alpha is the same person as the Omega. A form of reflexivity also holds for relative identity, but of a form weaker than the so-called strong reflexivity of absolute identity. Something is the same \( \phi \) as itself just in case it is \( \phi \). The Father is the same God as the Father, and the Father is the same person as the Father. But, since the Father is not a dog, the Father is not the same dog as the Father. Fido, on the other hand, is the same dog as Fido since Fido is a dog, while Fido is neither the same God nor the same person as Fido.

It is easy to see that Arius’s argument fails when the identity predicates are relativized.

1. \((\exists x)[Gx \& (y)(Gy \rightarrow y = x)]\)

2. \(Gf\)

4. \(f \neq s\)

\[\alpha. \quad [Ga \& (y)(Gy \rightarrow y = a)] \quad 1, \text{ existential instantiation.} \]

\[\beta. \quad (y)(Gy \rightarrow y = a) \quad \alpha, \text{ simplification.} \]

\[\gamma. \quad (Gf \rightarrow f = a) \quad \beta, \text{ universal instantiation.} \]

\[\delta. \quad f = a \quad 2, \gamma, \text{ modus ponens.} \]

\[\epsilon. \quad (Gs \rightarrow s = a) \quad \beta, \text{ universal instantiation.} \]

\[\zeta. \quad a \neq s \quad 4, \delta, \text{ substitutivity.} \]
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\( \eta. \ s \neq a \)

\( \xi, \ symmetry. \)

The inference to 3' is blocked. The rule of modus tollens no longer applies to \( \epsilon \) and \( \eta \), since \( \eta \) no longer contains the same predicate as the consequent of \( \epsilon \).

It may appear that Sabellianism also falls with our new symbolization of the identity predicates. For the analogue of the original Sabellian proof now fails. It establishes no more than the orthodox proposition that the Father is the same God as the Son:

**Sabellius's Argument**

1. \((\exists x)[Gx \& (y)(Gy \rightarrow y \equiv x)]\)

2. \(Gf\)

3. \(Gs\)

\(\alpha. \ [Ga \& (y)(Gy \rightarrow y \equiv a)]\)

\(\beta. \ (y)(Gy \rightarrow y \equiv a)\)

\(\gamma. \ (Gf \rightarrow f \equiv a)\)

\(\delta. \ f \equiv a\)

\(\epsilon. \ (Gs \rightarrow s \equiv a)\)

\(\zeta. \ s \equiv Sa\)

\(\eta. \ f \equiv s\)

This notwithstanding, Sabellius can have his heresy if he is bent on it. All he needs to do is to take advantage of the property of reflexivity and add the truth that the Son is the same person as the Son to the premises—in symbols,

\(\theta. \ s \equiv s\)

—in order to get the heretical conclusion that the Father is the same person as the Son. For

\(\iota. \ f \equiv s\)

follows from \(\eta \) and \(\theta \) by substitutivity.

A different response is needed to answer the Sabellian challenge: the rejection of the principle of Indiscernibility of Identicals and hence of the rule of Substitutivity of Identicals. Without the rule of substitutivity, Sabellius's argument cannot go through.

The benefits of rejecting the principle of indiscernibility and the rule of substitutivity extend beyond the conquest of Sabellianism. It allows us to conquer

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Patrispassianism, which is Sabellianism in a different guise. Consider the following argument in behalf of Patrispassianism:

1. There is only one God.
2. The Father is God.
3. The Son is God.
9. The Son suffered for our sins.
10. Therefore the Father suffered for our sins.

If we allow the rule of substitutivity, we lapse into heresy, even if we relativize identity:

1. \((\exists x)[Gx \& (y)(Gy \rightarrow y \equiv_G x)]\)
2. \(Gf\)
3. \(Gs\)
9. \(Ss\)
\(\alpha.\ [(Ga \& (y)(Gy \rightarrow y \equiv_G a))]\)
\(\beta.\ (y)(Gy \rightarrow y \equiv_G a)\)
\(\gamma.\ (Gf \rightarrow f \equiv_G a)\)
\(\delta.\ f \equiv_G a\)
\(\epsilon.\ (Gs \rightarrow s \equiv_G a)\)
\(\zeta.\ s \equiv_G a\)
\(\eta.\ a \equiv f\)
\(\theta.\ s \equiv f\)
10. \(Sf\)

Such recidivism cannot and need not be tolerated. Deprived of the rule of substitutivity, the inference to 10 is blocked.

It may seem radical to reject the principle of indiscernibility and the rule of substitutivity, but it is neither ad hoc nor unprecedented. Rejecting indiscernibility and substitutivity is part and parcel of the concept of relative identity. If \(x\) and \(y\) are the same \(P\) and \(x\) has a property \(Q\) in virtue of being \(P\), then we can conclude that \(y\) has \(Q\). As for precedents, a number of philosophers who did not defend relative identity have rejected the principle of indiscernibility on independent grounds. Aristotle for example, thought that one and the same referent could have different properties depending upon what "definition" was used to refer to the referent.\(^6\)

He was trying to outlaw the following sort of argument:

\(^6\) Sophistical Refutations 179a36; Physics 202b10 ff.
You do not know the man coming toward you.
The man coming toward you is your father.
Therefore you do not know your father.

Another version of the same sort of argument has bothered many contemporary philosophers:

George IV wanted to know whether Scott was Sir Walter.
Scott is Sir Walter.
Therefore George IV wanted to know whether Sir Walter was Sir Walter.

And Leibniz, of all people, did not accept, at least during his mature period, the law that bears his name.7 He believed that substitutivity of identicals had to be abandoned to avoid such invalid inferences as the following:

Socrates qua musical is talented.
Socrates qua snub-nosed is Socrates qua musical.
Therefore Socrates qua snub-nosed is talented.

More recently Leonard Linsky has recommended ejection of the Law of Substitutivity on quite general grounds.8 He shows that it permits such invalid inferences as contained in the following argument:

Hesperus is the evening star.
Hesperus is Phosphorus.
Therefore Phosphorus is the evening star.

While the premises are true, the conclusion is false. Phosphorus is not the evening star but the morning star.

IV

In closing I want to make two general points about philosophical theology.

1. The etymological roots of “theology,” θεολογία, are susceptible of many interpretations, some more and some less instructive. One of these interpretations is that theology is just talk about God. This is the broadest interpretation and also the one on the lowest level because it is undiscriminating; it excludes neither gossip nor slander about God nor, for that matter, blasphemy. In a way, this interpretation is accurate; it describes much of what passes for theology today. In another sense, it fails as a description. For it does not describe how one should talk about the subject matter of theology, God and things in relation to God. A definition of theology, then, should set its sights higher; it should be normative. Thus I propose that the etymology of “theology” be interpreted as the logic of talk about God. It was not until the end of the Middle Ages that this notion of theology began to decline. Moreover, this

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decline can also be correlated with certain medieval theologians who introduced a new logic, one that could not render orthodoxy consistent. Theology has still not recovered from this theological stand and its aftermath.

2. Historians of dogma often explain dogmatic statements as more mature and reflective statements of the vague and inchoate beliefs of earlier ages, and this maturity and reflectiveness is credited to the Holy Spirit. Although this is often merely a handy device for trying to justify historical aberrations, the principle is a sound one. I have tried to show how reflection on the logie of the Trinity leads to fuller and more precise statements of doctrine. I have tried to provide a rational reconstruction of how one might begin with a vague and inchoate statement of doctrine and ascend to more sophisticated and more coherent beliefs. I have not made many substantial historical claims about the development of Trinitarian dogma; I have talked from a different perspective, from the logical point of view.