THE purpose of this paper is to explain why the statement:

(1) "Nothing is red (all over) and green (all over) at the same time"

is analytic. The method will be to give two lines of argument, one informal and discursive, the other relatively formal and exact. Both will have essentially the same content. First, however, a brief statement of the significance of the problem.

Philosophers have contended that the qualities green and red are simple and unanalyzable. Hence the a priori character of (1) cannot be explained on the ground of its analyticity—it is claimed—since its denial does not violate the principle of contradiction. By way of contrast, consider:

(2) "All bachelors are unmarried,"
or, if you prefer:

(3) "Nothing is both a bachelor and married at the same time."

The concept bachelor is analyzable into unmarried man. Accordingly, (2) is analyzable into

(4) "All unmarried men are unmarried"

and the denial of (2), (3), or (4) is:

(5) "Someone is both married and not married,"

which violates the principle of contradiction.

The difficulty may also be stated in another way. An analytic sentence is one that can be reduced to a theorem of formal logic by putting synonyms for synonyms. If red and green are unanalyzable, then no replacement of their names by synonymous expressions in (1) will turn (1) into a theorem of formal logic. Hence (1) is not analytic, but it is a priori (as everyone admits).

Before replying to this charming argument for the synthetic a priori (and its neatness and force are undeniable), I should like

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to make a few preliminary comments, by way of orientation. In the first place, the notion of “simplicity,” unlike the notion of “analyzability,” seems to be psychological rather than logical. Empirical studies by no means indicate universal agreement on what characters in experience are “simple.” And yet, no criterion has been presented for simplicity (in the relevant sense) other than what people feel. However, the argument given above does not really depend on the simplicity of red and green, but merely on their unanalyzability; that is, on the fact that “red” and “green” do not possess any synonyms in English or in any other language, actual or merely possible, relevant to this problem (i.e., any synonyms whose substitution for “red” and “green” in (i) would transform it into a theorem of formal logic).

This is established, of course, by reflecting on the sense of “red” and “green” and by “seeing” that any definitions that would make “red” and “green” logically dependent (by making “red” and “green” synonymous with some logically dependent expressions $P$ and $Q$) would involve a violation of the intended meaning of these terms.

Thus, to refute this argument, it is necessary to find expressions $P$ and $Q$ which are synonymous with “red” and “green” respectively, and whose substitution for “red” and “green” in (i) turns it into a theorem of logic,—but we must be sure that $P$ is really synonymous with “red” in the intended sense, and likewise, “green” with $Q$. In short, we must show that the concepts of red and green involved really are analyzable; the question of simplicity we can drop as irrelevant.

A second remark: The assertion that the concepts red and green are “simple and unanalyzable” may sound very plausible when considered in isolation; but it becomes very paradoxical when we extend the field under consideration. Thus, if “red” and “green” are unanalyzable—what about “colored”? In some ways the notion of being colored seems to me simpler than the notion of being red. Thus it would strike me as very implausible if someone should argue that “colored” is merely short for “red or green or yellow or . . . or . . . .”

But it would be very odd (to say the least) if someone should maintain that
"Everything red is colored"
is also synthetic a priori.

Furthermore, what about the relational term: "indistinguishable (in color) from"? This is clearly related to the terms just mentioned; e.g., we have:

(7) "If \( X \) is indistinguishable in color from \( Y \) then \( X \) is colored,"

and if we mean by "red" an exact shade of that color, we have further:

(8) If \( X \) is red and \( Y \) is red, then \( X \) is indistinguishable from \( Y \) in color."

Yet the thesis that the relational term "indistinguishable in color from" is analyzable by means of (a finite number of) such nonrelational terms as "red" and "colored" is very dubious.

The point I am leading up to by way of all this is very simple: the color properties are not, after all, isolated; they form a system (or better, a continuum). It is not merely that there are reds and greens; there is the underlying property of being colored, of which red and green are specifications; and there are the varying relations of adjacency in this continuum. If any of the dependencies among the color properties and relations are synthetic a priori, then they all may well be: but this seems difficult to believe.

Turning now to the actual reply: I should like to begin by producing for consideration the sentence:

(9) "If \( A \) is not exactly the same color as \( B \), then if something is the same color as \( A \), then it is not exactly the same color as \( B \)."

I think there would be general agreement that (9) feels analytic; unlike (1). But at this point our "feelings" about these matters begin to conflict. For, suppose (9) is analytic. Then:

(10) "Nothing is the same color as \( A \) (all over) and the same color as \( B \) (all over) at the same time,"

which sounds like a genuine generalization, turns out to be equivalent to:

(11) "\( A \) and \( B \) are not exactly the same color"

which is merely "singular."
To show this by means of modern logic: symbolizing (9), (10), and (11) we get

for (9): \((x) (\sim \text{Ex}[A,B] \cup \text{Ex}[x,A] \cup \sim \text{Ex}[x,B])\)

for (10): \((x) \sim (\text{Ex}[x,A] \cdot \text{Ex}[x,B])\)

for (11): \(\sim \text{Ex}(A,B)\)

(Interpret "\text{Ex}[x,y]\) as "x is exactly the same color as y"

The third of these expressions is derivable from the first two quite easily, if one assumes that "Ex" is reflexive, and the second from the first and third. Thus the equivalence of (10) and (11) is indeed a consequence of (9)—and if (9) is analytic, then (10) and (11) are simply equivalent, i.e., each is a consequence of the other. Thus the apparently "universally valid" statement (10) is merely a disguised report about \(A\) and \(B\).

Of course there is a question as to the analyticity of (9). But I don’t think that anyone will maintain that being exactly the same color as is "simple and unanalyzable." In fact it is not even ostensively definable.

If we reflect on the relation "exactly the same color as" we find, in fact, that it is peculiarly difficult to pin down. It appears to have a close connection with the relation of indistinguishability in color from, which is the relation that one would communicate if one tried to give the meaning of "exactly the same color as" by pointing out examples. It is, however, distinct from this relation, since it is clearly transitive (which is all that (9) says) whereas "indistinguishable in color from" is nontransitive, since we could find a chain \(a_1, a_2, \ldots, a_{10}\), let us say, such that \(a_1\) is indistinguishable in color from \(a_2\), \(a_2\) is in turn indistinguishable in color from \(a_3, \ldots, a_9\) is indistinguishable in color from \(a_{10}\); and yet \(a_1\) is distinguishable in color from \(a_{10}\).

We will avoid for the moment the problem of defining "exactly the same color as"; let us try instead to put down all of its intuitive properties (which can later serve as criteria of adequacy for any definition). Only two properties appear clear to me:

I. "Ex" is an equivalence relation, i.e., it is transitive, reflexive, and symmetrical.

II. "Ex" is stronger than "indistinguishable from," i.e., if \(\text{Ex}(x,y)\), then \(x\) is indistinguishable in color from \(y\).
If we are willing to admit that I and II determine, completely or incompletely, what we mean by “exactly the same color,” then we shall have to admit that (g) is analytic, since it is equivalent to:

\[(12) \ (x) \ (Ex[x,A] \vee Ex[x,B] \vee Ex[A,B])\]

which formally expresses the transitivity of “Ex.” But more than this. It will follow now, not merely that (10) is equivalent to “A is not exactly the same color as B,” but, by virtue of II, that this latter assertion is implied by “A is distinguishable in color from B”;—and this, of course, is a mere observation report.

In short, the report proposition that A is distinguishable in color from B is stronger than the statement that A is not exactly the same color as B; and this latter statement is strictly equivalent to the “universal” judgment that nothing is the same color as A and the same color as B (all over) at the same time.

Or again, if I point to A and to B (let us suppose for the moment that A is red, and B green) and say “Nothing is the same color as A and the same color as B at the same time,”—this “apodictic” judgment is not a discovery of an “incompatibility” between the colors exemplified by A and B: it is a mere fragment of a report; it says less, logically speaking, than the straightforward report “A is distinguishable in color from B.”

This, I think, is all that there is to the “factual certainty” of (10). The sentence (10) implies the sentence “A is not exactly the same color as B”; in this sense it “contains” it, as a factual (and even a corrigible) element. But, except for this apparently trivial presupposition, (10) appears to be (a) universal, and (b) certain. The foregoing analysis is designed to show that the certainty and universality of (10) (relative to this assumption) is analytic. In a similar way, one could show that “Nothing is the color that A appears to be and the color that B appears to be (all over) at the same time”—which could have been considered instead of (10)—is equivalent to “A does not appear to be the same color as B.”

The problem is, how to apply our analysis of (10) to sentence (1). Suppose a speaker has in mind by “red” any shade of red whatsoever, and by “green” any shade of green whatsoever. Then he must be able to imagine at least two objects, A and B, such that (1) he would apply the term “red” (in that sense) to A and the term “green” (in that sense) to B; and (2) A and B are
distinguishable in color. Please note: I have not said that "he must never apply 'red' and 'green' (in whichever sense) to the same object." What I have said is that he must be able to imagine at least one object which is red but not green, and at least one object which is green but not red. Otherwise, he would simply be using the words "red" and "green" as synonyms (and this particular misuse would, of course, be easily detected). What this amounts to, of course, is that by "that shade of red" and "that shade of green" anyone speaking standard English must mean "exactly the same color as A" and "exactly the same color as B" where A and B must be so chosen or imagined as to be distinguishable in color. But if "that shade of red" and "that shade of green" are always the colors of objects which are distinguishable in color, then it is not a cause for wonder that "Nothing is that shade of red all over and also that shade of green all over at the same time" is always true, no matter who asserts it. And if it is true that no matter which shade of red and which shade of green we choose, nothing is both that shade of red and that shade of green, then it is true that "Nothing is both red and green" even if by "red" and "green" we mean not specific shades but broad classes of such shades.

One more remark before closing the informal part of our argument: the rule of usage described above does not, of course, determine what any particular speaker shall mean by "that shade of red" or "that shade of green" in any particular context; or even what he shall mean by the broader terms "red," "green," and "indistinguishable." Speaking scientifically rather than epistemologically (e.g., in terms of what physiology might disclose) I think it not impossible that someone's spectrum might be reversed, i.e., he might see blue where I see red and vice versa. Nevertheless, he would use the color terms "correctly" enough by ordinary standards, i.e., in conformity with such rules as that given above. This acknowledges what was sound in structuralism, as the doctrine was called which held that we can communicate the structure of our experience but not its felt quality.2

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2 R. Carnap, Der logische Aufbau der Welt (Berlin, 1928), pp. 11-21.
Now for the formal part of the argument. What I propose to do is to re-present the core of the foregoing informal argument as a sketch for the building up of the color concepts within a constructive system or formal language. Since this method of representing and consolidating a philosophic position is currently out of fashion, it seems desirable to begin with a remark explaining my reasons for preferring it.

Briefly, it appears to me that the most important disagreement among "analysts" today is between those who believe that there is a correct answer to such a question as the one we have been discussing, whether this is to be obtained by therapeutically dissolving the puzzle through the examination of usages, or through "straightforward analysis" of the problem; and those who feel, as I do, that there is never one final answer, but a variety of different answers based on different explications of the crucial concepts. On this latter view, a philosophic analysis merely presents one out of many possible reconstructions of a group of concepts; the aim of the analysis is to develop the theory of these concepts; and anyone who feels that his meaning for the concepts has not been clarified is invited to develop his own explication, and to discover how an alternative interpretation of the concepts would affect the theory.

In particular, what we have done informally in the preceding pages is to sketch a reconstruction of the color concepts, and to show how this reconstruction would affect the interpretation of some philosophically puzzling sentences. This reconstruction is certainly not the only possible one; it does, however, correspond fairly well to certain rules of English usage (that have been pointed out) and to what are for me the intuitive meanings of the concepts involved. Still, it seems important to point out that what has been presented is a reconstruction and not an attempted description of ordinary English usage. For this reason, the analysis will now be presented explicitly in constructional form.

The underlying theme of the construction is this: when we think of the color concepts, the most striking fact we observe is that they form a continuum. Likewise with the sounds, etc. This fact enables us to classify a new color we may never have encountered before as a color; it fits into one continuum but not into the other.
But to say that qualities form a continuum is to say that we can pass by imperceptible stages from one to the other; in the case of colors this "passing by imperceptible stages" is done by means of the relation of indistinguishability with respect to color. Thus this relation seems to "constitute" the color-continuum; and it is this relation (which will be symbolized as "Ind") which is taken as primitive.

Into our construction we shall wish to introduce names for specific shades, say \( F \) and \( G \). The general method for doing this has already been indicated; we define a specific shade as the color of some object (physical or phenomenal, depending on which form of language we have chosen). Intuitively, it does seem that whenever we speak or think of "that specific shade" we have some "object in mind"—there is no other way to "pin down" the shade. But this is merely an informal comment by way of supporting the following definition patterns:

(13) \( "F(x)" \) for \( \"Ex(x,A)\" \)
\( "G(x)" \) for \( \"Ex(x,B)\" \)

etc.

We can also define the general notion of a color:

(14) \( \"Col(F)\" \) for \( \"(\exists y) (x) [F(x) = Ex(x,y)]\" \)

but both of these definitions presuppose that we have already succeeded in defining "exactly the same color as" or "Ex." It is also clear from the informal part of the analysis that it is on our ability to frame a definition for this concept that our construction stands or falls. Fortunately, a definition appears to be possible: "\( x \) is exactly the same color as \( y \)" for "(for every \( z \)) \( z \) is indistinguishable in color from \( x \) if and only if \( z \) is indistinguishable in color from \( y \)."

This last proposal requires a little informal buttressing. The idea of the definition is suggested by this consideration: if \( A \) and \( B \) are not exactly the same color, then even if they are indistinguishable, something almost on the theoretical "threshold of distinguishability" with respect to \( A \) should be (in certain cases) indistinguishable from \( A \) but distinguishable from \( B \). On the other hand, if \( A \) and \( B \) are exactly the same color, then anything that is indistinguishable from \( A \) must also be indistinguishable from \( B \).
A somewhat more serious argument for the proposal is this: consider the conditions under which we would say that two objects are exactly the same color. These do not always coincide with the conditions under which we would say that they are indistinguishable, e.g., let $A$ and $B$ be indistinguishable, but suppose that $C$ is indistinguishable from $B$ but distinguishable from $A$. Then we would say that the color of $B$ is between that of $A$ and that of $C$ (not that it is exactly the same color as $A$ or $C$). In other words, that everything that is indistinguishable from $A$ be indistinguishable from $B$ (and vice versa) is a necessary condition for $A$ and $B$ being exactly the same color. But is it not also a sufficient condition? If $A$ and $B$ are not merely indistinguishable, but if in addition everything that is indistinguishable from $A$ is also indistinguishable from $B$ and vice versa,—then what possible further evidence could prove that $A$ and $B$ are not "really" the same color?

The most important consideration, however, is that the suggested definition satisfies the criteria of adequacy (I and II) that were put down before. These conditions formulate the intuitive properties of "$Ex$" (and I hasten to assure the suspicious reader that they were honestly laid down in advance of finding a definition, as bona fide criteria of adequacy must be); thus any definition which satisfies them must be considered as formulating an admissible concept of "exactly the same color."

If we are using a physicalistic language, then stronger concepts could, of course, be formulated. Thus the meaning of the expression "exactly the same color" might be specified in terms of performable laboratory operations, e.g., by means of reduction sentences. Another good method would be to construe this concept as a theoretical concept, to be implicitly defined by means of theoretical postulates (which would ultimately have to be interpreted, at least partially, in terms of "laboratory operations," or other observables). But a concept so introduced would presumably be even stronger than the one we have defined, for laboratory techniques would make possible even finer distinctions than are possible on the basis of unaugmented human ability to distinguish. Thus, such concepts would also satisfy our criteria of adequacy; which is all that is necessary for the present argument.
We pause to record our definition formally:\(^3\)

\[(15) \text{“} \text{Ex}(x,y)\text{” for “} (z) \text{ Ind}[z,x] = \text{Ind}[z,y] \text{“} \]

and also to postulate the fundamental properties of “Ind”—symmetry and reflexivity (but not transitivity!):

\[(16) \text{ a. } \text{Ind}(x,x) \]
\[(16) \text{ b. } (x,y) \text{ Ind}[x,y] = \text{Ind}[y,x] \]

The first of these postulates permits us to speak of Ind as holding or not holding between \(x\) and \(y\) even when \(x\) and \(y\) are not distinct (in this case we count it as trivially true); the second expresses the fact that Ind is fundamentally a characteristic of an unordered pair.

We are now in full possession of the results of the informal discussion. Thus, we have as a consequence of (15) (proofs will be omitted) the equivalence of (10) and (11); to say “Nothing is the same color as this and simultaneously the same color as that” is merely to assert that “this is not the same color as that.” Also, we have:

\[(17) \sim \text{Ind}(x,y) \supset \sim \text{Ex}(x,y) \]

or in words: “if \(x\) and \(y\) are distinguishable, then \(x\) and \(y\) are not exactly the same color”; and from this and the equivalence between (10) and (11) we have the main result of the informal discussion, namely:

\[(18) \sim \text{Ind}(x,y) \supset (z) \sim (\text{Ex}[z,x] \cdot \text{Ex}[z,y]) \]

which means, as was remarked, that the “apodictic” assertion that “Nothing is the same color as this and the same color as that at the same time” (we have confined the discussion to uniformly colored objects, for the sake of simplicity) is a weaker statement than the mere “protocol,” “this is distinguishable in color from that.”

In closing, I should like to sketch the continuation of the construction. The next step would be to define the second-level predicates “Red(F)” (for “\(F\) is a shade of red”) and “Grn(F)” (for “\(F\) is a shade of green”). In defining these predicates we are restricted by two postulates. The first formulates a feature of English usage pointed out in the informal discussion: Nothing

\(^3\) Goodman has earlier employed a similar definition; see N. Goodman, The Structure of Appearance (Cambridge, 1951), p. 221.
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can be classified as both a shade of red and a shade of green (i.e., “that shade of red” and “that shade of green” must never be used as synonyms). The second formulates the fact that red and green are intervals; if \( F \) and \( H \) are shades of red, and \( G \) is between \( F \) and \( H \) then \( G \) is a shade of red. The statement of the first postulate follows:

\[
(19) \quad (F) \sim (\text{Red}[F] \cdot \text{Grn}[F])
\]

The statement of the second postulate requires the definition of “BTW\((F,G,H)\)” (for “\( G \) is between \( F \) and \( G \) with respect to color,” as yellow, say, is between red and blue) in terms of “Ind.” The possibility of formulating a definition of “BTW” should be evident to any student of the classic constructional systems.⁴ The basic idea is to define “between” in terms of “directly between,” which is defined as follows:

\[
(20) \quad \text{“} x \text{ is directly between } y \text{ and } z \text{” for } \text{Ind}(x,y) \cdot \text{Ind}(x,z) \cdot \sim \text{Ind}(y,z). \]

Then the statement of the second postulate is:

\[
(21) \quad \text{Red}(F) \cdot \text{Red}(H) \cdot \text{BTW}(F,G,H) \supset \text{Red}(G),
\]

and similarly for the other colors. Finally we add postulates giving the relative positions of the colors on the spectrum, e.g.:

\[
(22) \quad \text{Red}(F) \cdot \text{Orange}(G) \cdot \text{Yellow}(H) \supset \text{BTW}(F,G,H),
\]

and further postulates specifying that there are at least two distinct shades of each color (lest someone think that “red” names a specific shade) and that there is no color between red and orange, etc.

It should be observed that these postulates define the logical structure of the color continuum, but not the actual location of the cutting points (or which end is red). Thus, anyone who uses this formal language according to the rules (of course it is not actually a language, but merely a sketch of a language) would mean by “red” a continuous series of shades, and by “orange” and “yellow,” other continuous series of shades (and by “orange” all the shades between “red” and “yellow”); but the rules do not determine beyond this which shades he would mean. The relation to natural language should be clear: the rules of natural language also determine that all the shades between red and yellow shall

⁴ E.g., Carnap, *op. cit.*, and Goodman, *op. cit.*
be classified as orange; but they do not determine such a thing as the place where orange stops and yellow begins.

Let us now consider the original problem: “Nothing can be both red and green.” Sometimes it is said that “the problem depends on red and green being ostensively defined” (so that they will be “simple and unanalyzable”). But, if they are literally defined by pointing, then “red” is being used temporarily as “the color of this” and “green” as “the color of that”—and this case has been already analyzed. Suppose, however, the assertion is meant as “Nothing can be a shade of red (in color) and a shade of green (in color) at the same time.” This symbolizes:

\[(23) \ (x) \sim (F[x] \cdot G[x] \cdot \text{Red}[F] \cdot \text{Grn}[G]).\]

This does not follow (as one might first think) from postulate (19) above. Postulate (19) merely tells us that if \(F\) is a shade of red and \(G\) a shade of green, then \(F\) is not the same shade as \(G\); but the statement we want is that if \(F\) is not the same shade as \(G\), then no \(x\) is both \(F\) and \(G\) in color; i.e., (23) would follow from (19) plus:

\[(24) \ (F) \ (G) \ (x) \ (\text{Col}[F] \cdot \text{Col}[G] \cdot F \neq G \supset \sim [F(x) \cdot G(x)]),\]

or “nothing is two different shades at once.”

This assertion constitutes the heart of our problem. If something were both red and green, it would presumably be two different shades at once; so the falsity of (1) entails the falsity of (24). Assertion (24), however, follows from the definition of “Col”; the usual logical rules for identity; and the crucial definition or explication:—that of “Ex.” The truth of (23) and (24) has thus been established by reference to the rules of our language; which makes them analytic in what I, for one, consider to be a perfectly clear sense. And the analyticity of (24), which is the heart of the matter, does not depend on any of our postulates (in case someone wonders whether too much might not have been smuggled in), but merely on the definitions plus logic (which is “good, old-fashioned” analyticity).

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