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Strict conditionals

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Conditionals as strict implications

- In this lecture, we discuss the thesis that natural language conditionals are *strict implications*, namely the thesis that the logical form ⌜*if* φ, *then* ψ⌝ is ⌜$\Box(\varphi \supset \psi)$⌝.
- Strict implication was introduced in 1912 by C. I. Lewis to analyze the ordinary meaning of "imply". The symbol he introduced later on for strict implication is this: $\prec$. Thus, we will use ⌜$\varphi \prec \psi$⌝ as short for ⌜$\Box(\varphi \supset \psi)$⌝
- (Lewis thought that axiomatic system S5, which corresponds to natural deduction system S5(NAT) we introduced for $\Box$, was inadequate to describe the ordinary meaning of "imply" and favoured weaker systems for necessity, but here we'll assume that S5(NAT) correctly characterizes the relevant notion of necessity).

A familiar problem

- As we already pointed out, it is not clear how a theory which assigns the same logical form both to indicative and counterfactual conditionals can account for the fact that (1) appears to be true, but (2) doesn't:

(1) If Oswald did not shoot Kennedy, someone else did.

(2) If Oswald had not shot Kennedy, someone else would have.

- Still one might wonder whether treating indicative conditionals as strict implications has any substantial advantage over treating them as material implications.
- Or one might wonder whether treating counterfactual conditionals as strict implications provides an adequate account of counterfactuals.

Indicative conditionals as strict implications

- As we have seen, the view that indicative conditionals are material implications runs into several difficulties.
- How does the view that indicative conditionals are strict implications fare in comparison?
- Let's see.

False antecedents and true consequents

- The thesis that indicative conditionals are material conditionals incorrectly predicts that the following conditionals should be true:

(3) If World War II ended in 1941 then gold is an acid.

(4) If New York is in the United States then World War II ended in 1945.

- This incorrect prediction depends on the fact that for material implications both the falsity of the antecedent and the truth of the consequent are sufficient to make the implication true.

Strict implication and paradoxes of material implication

- If natural language conditionals are strict conditionals, neither the falsity of the antecedent nor the truth of the consequent are sufficient to make the conditionals true.
- Indeed, the following claims are true:
  - $\psi \not\models_{LS5} \Box(\varphi \supset \psi)$, for every formula φ e ψ.
  - $\sim\varphi \not\models_{LS5} \Box(\varphi \supset \psi)$, for every formula φ e ψ.

Counter-models

- Indeed any model for LS5 which meets the conditions in 1-6 makes the premises in (a)-(b) true at $w0$ and the conclusion false at $w0$:
  (a) $q \not\models_{LS5} \Box(p \supset q)$
  (b) $\sim p \not\models_{LS5} \Box(p \supset q)$

1. $W = \{w0, w1\}$
2. $w0Rw0$, $w0Rw1$, $w1Rw1$, $w1Rw0$
3. $v(p, w0) = 0$
4. $v(q, w0) = 1$
5. $v(p, w1) = 1$
6. $v(q, w1) = 0$

The trouble with negation

- The thesis that indicative conditionals are material conditionals incorrectly predicts that arguments (5) and (6) are valid, since a material conditional is false exactly in case the antecedent is true and the consequent false:

(5) It is not true that if Mélenchon is the president of France, Macron won the elections. Thus, Mélenchon is the president of France.

(6) It is not the case that, if I go to the party tonight, I shall get drunk tonight. So, I shall not get drunk tonight.

Negation of strict conditionals

- The following claims are true:
  - $\sim\Box(\varphi \supset \psi) \not\models_{LS5} \varphi$, for every formula φ e ψ.
  - $\sim\Box(\varphi \supset \psi) \not\models_{LS5} \sim\psi$, for every formula φ e ψ.
- We can convince ourselves that they are true by reasoning thus: for $\ulcorner\sim \Box(\varphi \supset \psi)\urcorner$ to be true at a world w, it is neither required that φ be true at w nor required that $\ulcorner\sim \psi\urcorner$ be true at w, but it is sufficient that there is a world w' accessible from w at which φ and $\ulcorner\sim \psi\urcorner$ are true.
- Thus, the trouble with negation does not arise if we assume that natural language conditionals are strict conditionals: the falsity of the conditional requires neither that the antecedent be true nor that the consequent be false.

Countermodels

- Indeed, any model for LS5 which meets the conditions in 1-6 makes the premises in (c)-(d) true at $w0$ and the conclusion false at $w0$:

(c) $\sim\Box(p \supset q) \not\models_{LS5} p$
(d) $\sim\Box(p \supset q) \not\models_{LS5} \sim q$

1. $W = \{w0, w1\}$
2. $w0Rw0$, $w0Rw1$, $w1Rw1$, $w1Rw0$
3. $v(p, w0) = 0$
4. $v(q, w0) = 1$
5. $v(p, w1) = 1$
6. $v(q, w1) = 0$

The transitivity problem

- The thesis that indicative conditionals are material conditionals incorrectly predicts that argument (7) is valid, since the connective "⊃" is transitive:

(7) If Trump will not run for the 2024 elections, he will flee the country. If Trump is in jail, he will not run for the 2024 elections. Thus, if Trump is in jail, he will flee the country.

Transitivity and strict implication

- The connective of strict implication $\prec$ is transitive, namely the following is true:
  $\Box(\varphi \supset \psi), \Box(\psi \supset \xi) \models_{LS5} \Box(\varphi \supset \xi)$,
  for every formula φ, ψ e ξ.
- Thus the problem posed by the invalidity of (7) remains for the thesis that natural language conditionals are strict conditionals:

(7) If Trump will not run for the 2024 elections, he will flee the country. If Trump is in jail, he will not run for the 2024 elections. Thus, if Trump is in jail, he will flee the country.

- Here is a proof that the argument would be valid, if natural language conditionals were strict conditionals:

1.	$\Box(p \supset q)$	P
2.	$\Box(q \supset r)$	P
3.	prove $\Box(p \supset r)$	$\Box$I
4.	$p \supset q$	$\Box$E, 1
5.	$q \supset r$	$\Box$E, 2
6.	prove $p \supset r$	$\supset$I
7.	p	Ass
8.	q	$\supset$E, 4, 7
9.	r	$\supset$E, 5, 8

- p: Trump is in jail
- q: Trump will not run for the 2024 elections
- r: Trump will flee the country

The problem with strengthening of the antecedent

- The thesis that indicative conditionals are material conditionals incorrectly predicts that argument (8) is valid:

(8) If Holmes accepted the case, the case will be solved. Thus, if Holmes accepted the case and gave it up right after, the case will be solved.

Strengthening of the antecedent and strict implication

- The connective of strict implication $⥽$ validates strengthening of the antecedent, namely the following s true:
  - $\Box(\varphi \supset \psi) \models_{LS5} \Box((\varphi \wedge \xi) \supset \psi)$, for every formula φ, ψ e ξ.
- Thus the problem posed by the invalidity of (8) arises also for the thesis that natural language conditionals are strict conditionals:

(8) If Holmes accepted the case, the case will be solved. Thus, if Holmes accepted the case and gave it up right after, the case will be solved.

- The following proof shows that the argument would be valid, if natural language conditionals were strict conditionals:

1.	$\Box(p \supset q)$	P
2.	prove $\Box((p \wedge r) \supset q)$	$\Box$I
3.	$\Box(p \supset q)$	R,1
4.	$p \supset q$	$\Box$E, 3
5.	prove $(p \wedge r) \supset q$	$\supset$I
6.	$p \wedge r$	Ass
7.	p	$\wedge$E, 6
8.	q	$\supset$E, 4, 7

- p: Holmes accepted the case
- q: the case will be solved
- r: Holmes gave the case up right after

The problem with contraposition

- The thesis that indicative conditionals are material conditionals incorrectly predicts that argument (9) is valid:

(9) If it rains, it is not the case that will rain a lot. Therefore, if it rains a lot, it is not the case that it will rain.

Contraposition and strict implication

- It's easy to show that the connective of strict implication $\prec$3 validates the inferences by contraposition, namely the following is true:
  - $\Box(\varphi \supset\sim \psi) \models_{LS5} \Box(\psi \supset\sim \varphi)$, for every formula φ and ψ.
- Thus the problem posed by the invalidity of (9) also arises for the thesis that natural language conditionals are strict conditionals:

(9) If it rains, it is not the case that will rain a lot. Therefore, If it rains a lot, it is not the case that it will rain.

- The following proof shows that the argument would be valid, if natural language conditionals were strict conditionals:

1.	$\Box(p \supset\sim q)$	P
2.	prove $\Box(q \supset\sim p)$	$\Box$I
3.	$\Box(p \supset\sim q)$	R, 1
4.	$p \supset\sim q$	$\Box$E, 3
5.	prove $q \supset\sim p$	$\supset$I
6.	q	Ass
7.	prove $\sim\sim q$	$\sim$I
8.	$\sim q$	Ass
9.	q	R, 6
10.	$\sim p$	$\supset$E*, 7, 4

- p: it will rain
- q: it will rain a lot

Read's problem

- The thesis that indicative conditionals are material conditionals incorrectly predicts that sentence (10) is never false:

(10) If I am right, you are right or if you are right I am right.

- The problem, as Read observed, is that it's easy to imagine a situation in which disjunction (10) is false: think of a case in you and I hold incompatible views.

Strict implication and Read's problem

- The following is true:
  - $\not\models_{LS5}\ \Box(\varphi \supset \psi) \vee \Box(\psi \supset \varphi)$, per ogni formula φ e ψ.
- Thus, the problem posed by (10) does not arise for the thesis that natural language conditionals are strict conditionals:

(10) If I am right, you are right or if you are right I am right.

A counter-model

- We can see that $\Box(p \supset q) \vee \Box(q \supset p)$ is not a valid formula in LS5 by looking at the following model:
  - $W = \{w0, w1\}$
  - $w0Rw0$, $w0Rw1$, $w1Rw1$, $w1Rw0$
  - $\nu(p, w0) = 1$
  - $\nu(q, w0) = 0$
  - $\nu(p, w1) = 0$
  - $\nu(q, w1) = 1$
- Indeed, formula $\Box(p \supset q) \vee \Box(q \supset p)$ is false at $w0$.

The paradoxes of strict implication

- A material conditional is true if the antecedent is false or the consequent is true.
- For strict conditionals, as we saw, this is not the case.
- However, strict conditionals are true if the antecedent is necessarily false or the consequent is necessarily true (these facts are called "the paradoxes of strict implication"):
  - $\Box\psi \models_{LS5} \Box(\varphi \supset \psi)$, for every formula φ and ψ.
  - $\sim\Diamond\varphi \models_{LS5} \Box(\varphi \supset \psi)$, for every formula φ and ψ.
- Here's the reason: If $\ulcorner\Box\psi\urcorner$ is true at a world w, then ψ is true at every world accessible from w, thus $\ulcorner\varphi \supset \psi\urcorner$ is true at every world accessible from w, thus $\ulcorner\Box(\varphi \supset \psi)\urcorner$ is true at w. And if $\ulcorner\sim\Diamond\varphi\urcorner$ is true at a world w, then φ is false at every world accessible from w, thus $\ulcorner\varphi \supset \psi\urcorner$ is true at every world accessible from w, thus $\ulcorner\Box(\varphi \supset \psi)\urcorner$ is true at w.

Some consequences

- Thus, the thesis that natural language conditionals are strict conditionals, while it avoids the incorrect prediction that (3) and (4) are true, it incorrectly predicts that (11) and (12) are true:

(3) If World War II ended in 1941 then gold is an acid.

(4) If New York is in the United States then World War II ended in 1945.

(11) If Rome is in Italy, there is an infinity of natural numbers.

(12) If there is no infinity of natural numbers, Paris is in Italy.

- Conditionals (11) and (12) are true, if they are strict, since the consequent of (11) is necessarily true and the antecedent of (12) is necessarily false.

A final problem

- A final problem we mention for the thesis that indicative conditionals are strict conditionals has to do with the conditional in (1):

  (1) If Oswald did not shoot Kennedy, someone else did.

- As we pointed out, this conditional seems to be true, since we know that Kennedy was shot.
- However, if (1) were a strict conditional, we should expect it to be false, since there are possible worlds in which Oswald did not shoot Kennedy and no one did.
- What (1) shows is that requiring that an indicative conditional is true exactly in case the corresponding material conditional is necessarily true won't do.

Strict implication and counterfactuals

- The thesis that indicative conditionals are strict conditionals does not seem to have substantial advantages over the thesis that indicative conditionals are material conditionals.
- Does strict implication fare better with respect to counterfactual conditionals? Can we adequately describe the truth conditions of counterfactuals conditionals by assuming that they are material implications?

A bit of good news

- As we pointed out, if we assumed that counterfactual conditionals were material conditionals, we should expect them to be always true, since we take for granted that their antecedent is false.
- Clearly this is an unwelcome prediction, since some counterfactuals are clearly false: it is false that if Caesar had not gone to the Senate on the Ides of March he would have died there.
- The view that counterfactual conditionals are strict conditionals does not run into the same problem, since, as we saw, the falsity of the antecedent is not sufficient to make a strict conditional true.

Some bad news

- One problem with the view that counterfactuals are strict conditionals is that this view seems unable to distinguish true counterfactuals like (13) from false counterfactuals like (14) (the example is by Paolo Casalegno):

  (13) If Kant had died in 1778, the *Critique of Pure Reason* would have remained unfinished.

  (14) If Marlene Dietrich had become a nun, the *Critique of Pure Reason* would have remained unfinished.

- Kant published the *Critique of Pure Reason* in 1781 and it took him ten years to write it. So, in 1778 the *Critique of Pure Reason* was still incomplete and, if he had died in 1778, it would have remained so. Thus, (13) is clearly true.
- But Dietrich's becoming a nun could not have had any impact on Kant's writing the *Critique*. Thus, (14) is clearly false.
- The problem is that, if counterfactual conditionals were strict conditionals, we should expect (13) to be false. Indeed, if (13) were a strict conditional, there should be no possible world in which Kant dies in 1778 and the *Critique of Pure Reason* is completed. Clearly this not the case: there are possible worlds in which Kant publishes the *Critique of Pure Reason* in 1777.

More bad news

- Moreover, as we saw, strict implication validates transitivity, strengthening of the antecedent, and contraposition. But these argument forms seem to be invalid also for counterfactual conditionals as (15), (16), and (17) show:

(15) If J. Edgar Hoover were today a communist, then he would be a traitor. If J. Edgar Hoover had been born a Russian, then he would today be a communist. Therefore, if J. Edgar Hoover had been born a Russian, he would be a traitor.

(16) If this match had been struck, it would have lit. Therefore, if this match had been soaked in water overnight and it had been struck, it would have lit.

(17) If the U.S. had halted the bombing, then North Vietnam would not have agreed to negotiate. Therefore, if North Vietnam had agreed to negotiate, then the U.S. would not have halted the bombing.

Counterfactuals with impossible antecedents

- Finally, notice that, if counterfactual conditionals were strict conditionals, we should expect all counterfactuals with impossible antecedents to be true, since the corresponding material conditional is true in every possible world, thus necessarily true.
- However, this prediction is clearly incorrect. Indeed, while counterfactual (18) is true, counterfactuals (19) is false, although the antecedent is impossible (the examples are due to Graham Priest 2008):

(18) If Hobbes had squared the circle, he would have become a very famous mathematician.

(19) If you had squared the circle, I would have given you my life's savings.

Demise of a theory?

- The view that indicative conditionals are strict implications does not seem to have substantial advantages over the thesis that indicative conditionals are material implications.
- Moreover, the view that counterfactual conditionals are strict implications seems to run into several problems as well.
- Does this mean that we should forget about strict conditionals?
- Perhaps not. Recent work by von Fintel (2001) suggests a way of making the strict analysis of counterfactuals more adequate.

The role of accessibility

- While we won't present more recent versions of the strict analysis of counterfactuals here, we can perhaps get an idea of their starting point by means of the following considerations.
- Notice that the problem with pointed out for the strict analysis of (13) arises to some extent from our assumption that the accessibility relation is universal (as required by LS5):

(13) If Kant had died in 1778, the *Critique of Pure Reason* would have remained unfinished.

- If every possible world is accessible form every possible world, then (13) is false because there are worlds accessible from our world in which Kant publishes the *Critique of Pure Reason* in 1777.
- But perhaps, when we regard (13) as true, we do not assume that such worlds are accessible. So, perhaps we should not assume that the accessibility relation is universal. Indeed, Warmbrod (1981) suggests that counterfactuals are strict conditionals where the accessibility relation is contextually determined. This is also where von Fintel's recent strict analysis of counterfactuals starts from.

Summing up

- We discussed the view that natural language conditionals are strict conditionals.
- We saw some problems for the strict analysis of indicative conditionals. And we also saw some problems for the strict analysis of counterfactuals.
- The analysis has been recently revived by von Fintel's work.