Università degli Studi di Milano

The three barbers revisited

Sandro Zucchi

Siena, November 30^{th}, 2022

A challenge on the way to the barber shop

"What, nothing to do?" said Uncle Jim. "Then come along with me down to Allen's. And you can just take a turn while I get myself shaved."[. . .]
After a bit, Uncle Jim began again, just as we came in sight of the barber's. "I only hope Carr will be at home," he said. "Brown's so clumsy. And Allen's hand has been shaky ever since he had that fever."
"Carr's certain to be in", said Uncle Joe.
"I'll bet you sixpence he isn't!" said I.
"Keep your bets for your betters," said Uncle Joe. "I mean"–he hurried on, seeing by the grin on my face what a slip he'd made–"I mean that I can prove it, logically. It isn't a matter of chance."
"Prove it logically!" sneered Uncle Jim. "Fire away, then! I defy you to do it!"

Uncle Joe accepts the challenge

. . . "Do you grant me that, if Carr is out, it follows that if Allen is out Brown must be in?"
"Of course he must," said Uncle Jim; "or there'd be nobody to mind the shop."[. . .]
"We have now to consider another *Hypothetical. What was that you told me yesterday about Allen?"*
"I told you", said Uncle Jim, "that ever since he had that fever he's been so nervous about going out alone, he always takes Brown with him."
"Just so," said Uncle Joe. "Then the Hypothetical 'if Allen is out Brown is out' is always in force, isn't it?"
"I suppose so," said Uncle Jim. (He seemed to be getting a little nervous, himself, now.)
"Then, if Carr is out, we have two *Hypotheticals, 'if Allen is out Brown is* in*' and 'If Allen is out Brown is* out*,' in force at once. And* two *incompatible *Hypotheticals, mark you! They can't* possibly *be true together!"*
"Can't they?" said Uncle Jim.
"How can they?" said Uncle Joe. How can one and the same protasis *prove two contradictory* apodoses*? You grant that the two* apodoses*, 'Brown is* in*' and 'Brown is* out*',* are *contradictory, I suppose ?"*
"Yes, I grant that," said Uncle Jim.
"Then I may sum up," said Uncle Joe. "If Carr is out, these two Hypotheticals are true together. And we know that they cannot *be true together. Which is absurd. Therefore Carr* cannot *be out. There's a nice* Reductio ad Absurdum *for you!"*
(L. Carroll, July 1894, "A Logical Paradox", 3:11, Mind, pp. 436-38)

Uncle Joe's argument

- Uncle Joe's argument consists of two premises and a conclusion:

  Premise one: if Carr is out, then if Allen is out Brown is in.
  Premise two: if Allen is out Brown is out.
  Conclusion: Carr is in.

- Uncle Joe and Uncle Jim agree that both premises are true in virtue of facts (a) and (b):
  (a) at least one of the barbers must be in the shop,
  (b) Allen always takes Brown with him when he goes out.
- Moreover, Uncle Joe claims that, given Premise one and Premise two, he can prove the conclusion that Carr is in by *reductio ad absurdum*.

Uncle Joe's proof

Premise one: if Carr is out, then if Allen is out Brown is in.

Premise two: if Allen is out Brown is out.

Conclusion: Carr is in.

Proof:

1. Carr is out [Assumption of the proof by *reductio*].
2. If Allen is out Brown is in [by *modus ponens*, from 1 and Premise one].
3. If Allen is out Brown is in and if Allen is out Brown is out [from 2 and Premise two, by conjunction introduction].
4. ~(If Allen is out Brown is in and if Allen is out Brown is out) [since it is impossible that the conjunction is true].

What goes wrong?

- Clearly, there must be something wrong in Uncle Joe's reasoning.
- Consider facts in (a) and (b) again:
  - (a) at least one of the barbers must be in the shop,
  - (b) Allen always takes Brown with him when he goes out.
- These facts are compatible with the possibility that Carr is out and Allen and Brown are in, or that Carr and Brown are out and Allen is in.
- So, Uncle Joe should *not* be able to conclude that Carr is in on the basis of facts (a) and (b).
- But what is wrong in Uncle Joe's reasoning precisely?

Answering a call for help

- In a final footnote to his paper, Carroll says: "I greatly hope that some of the readers of MIND who take an interest in logic will assist in clearing up these curious difficulties".
- Since July 1894 (the date of publication of Carroll's logical paradox), the discussion about conditionals has moved forward considerably.
- In this talk, I'll discuss some ways to help Lewis Carroll also in the light of what has come after in the literature on conditionals.

First help

- The first help for Carroll came few months after the publication of his paper on the paradox.
- In October 1894, a reply was published in *Mind* by W. E. Johnson, which proposed a way out of the logical paradox by appealing to the theory that indicative conditionals are material implications: they are false only in case the antecedent is true and the consequent is false.
- In 1903, B. Russell also claimed that this theory solves Carroll's paradox.
- The view that indicative conditionals are material implications goes back to Philo of Megara (c. 400s BCE) and has been advocated in the contemporary philosophical literature by Lewis (1976, 1986) and Jackson (1979, 1981, 1987). More recently, it has been revived by Williamson (2020).

Johnson on Carroll's logical paradox

. . . The two disputants may agree in expressing the problem in the following form:—

Principal Antecedent: *Carr is out.*
Principal Consequents: *If Allen is out, Brown is in;*
If Allen is out, Brown is out.

Uncle Joe uses the general method of the reductio ad absurdum, *for he disproves the principal antecedent by maintaining that the consequents to which it leads are incompatible. But in reality the two sub-hypotheticals which form his principal consequents are* not *incompatible. For in saying that two propositions are incompatible we mean that their combination involves a logical impossibility. Now the combination of these sub-hypotheticals does* not *involve any impossibility, but involves merely the denial that* Allen can be out. *[. . .]*
As regards the hypothetical of the general form "If A then B" we have interpreted this as the mere denial of the conjunction "A true and B false". The consistent application of this interpretation yields the above solution to the whole problem
(W. E. Johnson, October 1894, "A Logical Paradox", 3:12, Mind, p. 583)

Russell on Carroll's logical paradox

The principle that false propositions [materially] imply all propositions solves Lewis Carroll's logical paradox in Mind, *N. S. No. 11 (1894). The assertion made in that paradox is that, if p, q, r be propositions, and q implies r, while p implies that q implies not-r, then p must be false, on the supposed ground that "q implies r" and "q implies not-r" are incompatible. But in virtue of our definition of negation, if q be false both these implications will hold: the two together, in fact, whatever proposition r may be, are equivalent to not-q. Thus the only inference warranted by Lewis Carroll's premisses is that if p be true, q must be false, i.e. that p implies not-q; and this is the conclusion, oddly enough, which common sense would have drawn in the particular case which he discusses.*
(B. Russell, *The principles of mathematics*, 1903, sect. 19, ft. 1).

The Philonian narrative

- I'll call the narrative proposed by Johnson and Russell *the Philonian narrative*.
- This narrative consists of the following claims:
  1. Indicative conditionals are material implications: they are false only in case the antecedent is true and the consequent is false.
  2. Since indicative conditionals are material implications, conditionals (c1)-(c2) can be true together if the antecedent is false:
     (c1) if Allen is out, Brown is in
     (c2) if Allen is out, Brown is out
  3. The last step in Uncle Joe's proof should be rejected, since it is based on the claim that (c1)-(c2) cannot be true together.

The invalid step

Premise one: if Carr is out, then if Allen is out Brown is in.
Premise two: if Allen is out Brown is out.
Conclusion: Carr is in.
Proof:
1. Carr is out [Assumption of the proof by *reductio*].
2. If Allen is out Brown is in [by *modus ponens*, from 1 and Premise one].
3. If Allen is out Brown is in and if Allen is out Brown is out [from 2 and Premise two, by conjunction introduction].
4. ~(If Allen is out Brown is in and if Allen is out Brown is out) [since it is impossible that the conjunction is true].

Notice

- If indicative conditionals are material implications, not only Uncle Joe failed to prove *by reductio* that the conclusion follows from the premises, but *no proof that the conclusion follows from the premises can be given*.

  Premise one: if Carr is out, then if Allen is out Brown is in.
  Premise two: if Allen is out Brown is out.
  Conclusion: Carr is in.

- Indeed, if indicative conditionals are material implications, the premises of the argument can be true and the conclusion false.

Counter-models

p: Carr is out
q: Allen is out
r: Brown is out

1. if Carr is out, then if Allen is out Brown is in.
   $p \supset (q \supset \sim r)$
2. if Allen is out Brown is out.
   $q \supset r$
3. Carr is in.
   $\sim p$

The formulae in 1-2 are true and the formula in 3 is false under valuations v_1 and v_2:

$$\begin{array}{ll} v_1(p) = 1 & v_2(p) = 1 \\ v_1(q) = 0 & v_2(q) = 0 \\ v_1(r) = 1 & v_2(r) = 0 \\ \ldots & \ldots \end{array}$$

A trade-off with common sense

- The Philonian narrative allows us to reject Uncle Joe's unwarranted conclusion that Carr is in by rejecting step 4 of his proof:
  4. ~(If Allen is out Brown is in and if Allen is out Brown is out) [since it is impossible that the conjunction is true].
- Yet, the Philonian narrative achieves this result at a cost, since it rejects common sense inference (1):

(1) If Allen is out, Brown is out. Therefore, it is false that if Allen is out, Brown is in.

Not general enough

- Moreover, the Philonian narrative is not general enough, in view of the following subjunctive version of Carroll's argument:

  Premise one: if Carr were out, then if Allen were out Brown would be in.
  Premise two: if Allen were out Brown would be out.
  Conclusion: Carr is in.

- Premise one is true in virtue of fact (a), Premise two is true in virtue of fact (b):
  (a) at least one of the barbers must be in the shop,
  (b) Allen always takes Brown with him when he goes out.

  Yet, one should not be able to conclude that Carr is in from these premises. Thus, something must be wrong with the subjunctive version of the argument as well.
- The problem is that the Philonian narrative says nothing about the subjunctive version, since the premises are not indicative conditionals.
- (While, as we saw, the view that indicative conditionals are material conditionals is still alive today, even the supporters of this view concede that subjunctive conditionals are not material conditionals).

Carroll's paradox and minimal change semantics

- Now, I turn to a different kind of narrative, by which both indicative and subjunctive conditionals are *intensional*.
- In particular, according to the view I'll consider, in order to determine whether a conditional is true we have to look at the world(s) at which the antecedent is true that differs (differ) minimally from the real world.

Stalnaker's abstract semantics

- Stalnaker (1968, 1975) has claimed that indicative and subjunctive conditionals share the same *abstract semantics*:

  ⌜*if* φ, *then* ψ⌝ is true at a possible world w just in case:

  i. There is no (accessible) possible world at which φ is true, or
  ii. ψ is true at $f(\varphi, w)$, where f is a contextually determined function which selects the world at which φ is true that minimally differs from w.
- Let's see how this analysis of conditionals fares with respect to Carroll's paradox.

Some valid inferences

- Stalnaker's abstract semantics for natural language conditionals validates both *modus ponens* and *non-vacuous truth*:

  Modus ponens: *if* φ, *then* ψ, $\varphi \vDash \psi$
  Non-vacuous truth: $\diamond\varphi$, *if* φ, *then* $\psi \vDash \sim$(*if* φ *then* $\sim\psi$)
- It validates *modus ponens* for this reason. Suppose ⌜*if* φ, *then* ψ⌝ and φ are true at a world w. Then, ψ is true at the world minimally different from w at which φ is true. Since φ is true at w, the world minimally different from w at which φ is true is w itself. So, ψ is true at w.
- It validates *non-vacuous truth* for this reason. Suppose ⌜$\diamond\varphi$⌝ and ⌜*if* φ *then* ψ⌝ are true at a world w. Since φ is true at some possible world, it follows that ψ is true at the world w' minimally different from w in which φ is true. Thus, ⌜$\sim\psi$⌝ is false at w'. Thus, ⌜*if* φ, *then* $\sim\psi$⌝ is false at w. Thus, ⌜$\sim$(*if* φ *then* $\sim\psi$)⌝ is true at w.

A prediction

- Since Stalnaker's abstract semantics validates *modus ponens* and *non-vacuous truth*, Uncle Joe's argument is predicted to be valid (assuming conjunction introduction is valid):

  Premise one: if Carr is out, then if Allen is out Brown is in.
  Premise two: if Allen is out Brown is out.
  Conclusion: Carr is in.
  Proof:
  1. Carr is out [Assumption of the proof by *reductio*].
  2. If Allen is out Brown is in [by *modus ponens*, from 1 and Premise one].
  3. If Allen is out Brown is in and if Allen is out Brown is out [from 2 and Premise two, by conjunction introduction].
  4. ∼(If Allen is out Brown is in and if Allen is out Brown is out) [since it is impossible that the conjunction is true].
- In particular, by *non-vacuous truth*, "∼(if Allen is out, Brown is out)" follows from "if Allen is out, Brown is in" (and vice versa). Thus, conditionals (c1)-(c2), as Uncle Joe claims, cannot be true together:

  (c1) if Allen is out, Brown is in
  (c2) if Allen is out, Brown is out

Challenging the soundness

- Since Uncle Joe's argument is valid by Stalnaker's semantics, the only way for Stalnaker to reject the argument is to reject one of the premises.
- In the circumstances described in the story, (a)-(b) hold:
  (a) at least one barber must be in the shop
  (b) Allen always takes Brown with him when he goes out.
- Since the world *w* in which Allen is out which differs minimally from these circumstances is presumably still a world in which (b) holds, Brown is also out in *w*, and thus Premise 2 is true by Stalnaker's semantics:

  Premise two: if Allen is out Brown is out.
- On the other hand, Stalnaker might argue that Premise one is not true, since in evaluating the embedded conditional "if Allen is out, Brown is in", we should keep assumptions (a)-(b) fixed. If we do, then by Stalnaker's semantics we must conclude that Premise one is false, since the world in which Allen is out and (a)-(b) hold which differs minimally from a world in which Carr is out is a world in which Carr is in and Brown and Allen are out.

  Premise one: if Carr is out, then if Allen is out Brown is in.

The Stalnakerian narrative

- To sum up, the Stalnakerian narrative about Carroll's paradox consists of the following claims:
  1. Natural language conditionals are minimal change conditionals: they are true if their consequent is true in the world in which the antecedent is true that differs minimally from the actual world.
  2. Uncle Joe's argument is valid, since *modus ponens*, *non-vacuous truth*, and conjunction introduction are valid inferences.
  3. But Uncle Joe's argument is unsound, because Premise one is false:

     Premise one: if Carr is out, then if Allen is out Brown is in.

A possible alternative

- Before I go on to raise a problem for the Stalnakerian narrative, let me briefly comment on a possible alternative to the two narratives considered so far.
- The alternative is based on Lewis's (1973, 1976) account of natural language conditionals.

The Lewisian narrative

- The *Lewisian narrative* is a *mixed* narrative. It consists of these claims:
  1. Indicative conditionals are material conditionals.
  2. Subjunctive conditionals have the following truth-conditions: $\ulcorner$*if* φ, *then* $\psi\urcorner$ is true at a possible world *w* just in case:
     i. There is no accessible world at which φ is true, or
     ii. some accessibile world in which φ and ψ are true differs less from *w* than any world in which φ is true and ψ is false.
- Uncle Joe's argument is *invalid*, since conditionals (c1)-(c2) can be true together if the antecedent is false:

  (c1) if Allen is out, Brown is in
  (c2) if Allen is out, Brown is out
- The subjunctive version of Uncle Joe's argument is *valid but unsound*, since Lewis's semantics for subjunctives predicts that Premise one is false:

  Premise one: if Carr were out, then if Allen were out Brown would be in.

The reason for the prediction

- By Lewis's semantics for subjunctive conditionals, Premise one of the subjunctive argument should be false essentially for the same reason indicative Premise one turns out to be false by Stalnaker's semantics:

  Premise one: if Carr were out, then if Allen were out Brown would be in.

- The worlds in which Carr is out that are closest to a world in which (a)-(b) hold are also worlds in which (a)-(b) hold:
  (a) at least one barber must be in the shop
  (b) Allen always takes Brown with him when he goes out.
- But the worlds in which Allen is out which are closest to the worlds in which Carr is out and (a)-(b) hold are worlds in which Carr is in and Allen and Brown are out.
- Thus, subjunctive Premise one is predicted to be false.

Non-vacuous truth

- Notice that Lewis's semantics for subjunctive conditionals, like Stalnaker's, validates *non-vacuous truth* (as well as *modus ponens*):

  Non-vacuous truth: $\diamond\varphi$, *if* φ, *then* $\psi \vDash \sim$(*if* φ *then* $\sim\psi$)

- Indeed, suppose $\ulcorner\diamond\varphi\urcorner$ and $\ulcorner$*if* φ *then* $\psi\urcorner$ are true at a world w. Since φ is true at some accessible possible world, it follows some accessibile world w' in which φ and ψ are true differs less from w than any world in which φ is true and ψ is false. Thus, $\ulcorner\sim\psi\urcorner$ is false at w'. Thus, $\ulcorner$*if* φ, *then* $\sim\psi\urcorner$ is false at w. Thus, $\ulcorner\sim$(*if* φ *then* $\sim\psi\urcorner$ is true at w.

A problem for the Lewisian narrative

- Since for Lewis indicative conditionals are material conditionals, he must reject the validity of step 4 of Uncle Joe's argument:
  4. ~(If Allen is out Brown is in and if Allen is out Brown is out) [since it is impossible that the conjunction is true]
- On the other hand, since his account of subjunctive conditionals supports the validity of *non-vacuous truth*, he must accept the subjunctive version of 4 as valid:
  4′. ~(If Allen were out Brown would be in and if Allen were out Brown would be out)
- I take it that the evidence fails to support this distinction. There is no reason to reject *non-vacuous truth* for indicative (1) and accept it for subjunctive (1'):

(1) If Allen is out, Brown is out. Therefore, it is false that if Allen is out, Brown is in.

(1') If Allen were out, Brown would be out. Therefore, it is false that if Allen were out, Brown would be in.

A problem for the Stalnakerian narrative

- Let's now go back to the Stalnakerian narrative. Some evidence for the narrative comes from the fact that one might object to indicative Premise one as in the following dialogue:

  Speaker A: if Carr is out, then if Allen is out, Brown is in.
  Speaker B: Wait a minute, I thought Carr and Allen cannot be out together! If Allen is out, Brown must be out and Carr must be in.

- Notice, however, that subjunctive Premise one cannot be challenged in the same way, as the following reply shows:

  Speaker A (replying to B): Ok, but suppose Allen *were* out, if Carr were out. Then, Brown would have to be in (otherwise there would be nobody to mind shop).

- If A's reply to B is correct, subjunctive Premise one is true, but then it is not obvious how Stalnaker can reject the subjunctive version of the argument.

  Premise one: if Carr were out, then if Allen were out Brown would be in.

Taking stock

- So far, I have presented three different narratives about Carroll's paradox:
  - the Philonian narrative,
  - the Stalnakerian narrative,
  - the Lewisian narrative.
- The Philonian narrative claims that Uncle Joe's argument is not valid, by rejecting the common sense inference:

  if Allen is out, Brown is out.
  Therefore, it is false that if Allen is out, Brown is in.
- The Stalnakerian narrative claims that Uncle Joe's argument is valid, but Premise one is false, thus the argument is unsound.
- Both narratives have difficulties in accounting for the failure of the subjunctive version of Uncle Joe's argument.
- Finally, the Lewisian narrative agrees with the Philonian narrative's claim that Uncle Joe's argument is invalid, but agrees with the Stalnakerian narrative's claim that the subjunctive version of the argument is valid. The problem is that there is no empirical evidence for this difference in validity.

A mere verbal fallacy

- In the April 1905 issue of *Mind*, a note on Lewis Carroll's logical paradox appeared with the signature "W."
- In the note, the author makes the following observation concerning Uncle Joe's argument (coloring is mine):

  The fallacious reductio ad absurdum *argument starts from the proposition. . .*

  (v.) 'If Carr is out, then if Allen is out, Brown is in.'

  Now this means–

  'If Carr is out and Allen is out, Brown is in' [. . .]

  The false reductio ad absurdum *depends on a mistake about the meaning of proposition (v.) : 'If Carr is out, then, if Allen is out Brown is in', which is interpreted to mean that the proposition 'If Allen is out Brown is in' is a consequent of the assumption 'Carr is out'. This is a mere verbal fallacy.*

The suggestion by an execrably bad logician

- Apparently, the author of the April 1905 note in *Mind* is John Cook Wilson (1849-1915), an Oxford philosopher called by Geach (1978) "an execrably bad logician".
- Cook Wilson's claim is that the source of trouble in Uncle Joe's argument is the assumption that in Premise one the conditional "if Allen is out Brown is in" is the consequent of the supposition that Carr is out.

  Premise one: if Carr is out, then if Allen is out Brown is in.
- According to Cook Wilson, this assumption is "a mere verbal fallacy". He seems to suggest that the surface form of Premise one is deceiving, since the underlying logical form of Premise one is (2):

  (2) if Carr is out and Allen is out, Brown is in.
- The last narrative I'm going to present, the one (a version of which) I endorse, is based on the execrable logician's suggestion.

Back to subjunctive Premise one

- Suppose that conditionals have Stalnaker's abstract semantics.
- Suppose, moreover, that, as the bad logician suggests, the logical form of (a) is (b):

  (a) if φ, then if χ, then ψ
  (b) if φ and χ, then ψ
- By the latter assumption, subjunctive Premise one has logical form (3):

  Premise one: if Carr were out, then if Allen were out Brown would be in.

  (3) if Carr were out and Allen were out, Brown would be in.
- By Stalnaker's semantics, (3) is true, since the world w in which Carr and Allen are out that is closer to a world in which (a)-(b) hold is a world in which (a) still holds, thus a world in which Brown is in.

  (a) at least one barber must be in the shop
  (b) Allen always takes Brown with him when he goes out.
- Thus, subjunctive Premise one is predicted to be true, in accord with intuition.

The invalid step

- What is wrong both in the proof of the indicative argument and in the proof of the subjunctive argument is now step 2:

  Premise one: if Carr is/were out, then if Allen is/were out Brown is/would be in.
  Premise two: if Allen is/were out Brown is/would be out.
  Conclusion: Carr is in.
  Proof:
  1. Suppose Carr is/were out [Assumption of the proof by *reductio*].
  2. If Allen is/were out Brown is/would be in [by *modus ponens*, from 1 and Premise one].

  ...

- Step 2 is invalid, since the logical form of Premise one is (4) and thus step 2, despite appearances, does not follow by *modus ponens*:

  (4) if Carr is/were out and Allen is/were out Brown is/would be in.

The oddness of the indicative premise

- In discussing the Stalnakerian narrative, we conceded that there is something odd in asserting indicative Premise one against the background of (a) and (b):

  Premise one: if Carr is out, then if Allen is out Brown is be in.

  (a) at least one barber must be in the shop
  (b) Allen always takes Brown with him when he goes out.

- As we saw, one might object to the premise as in the following dialogue:

  Speaker A: if Carr is out, then if Allen is out, Brown is in.
  Speaker B: Wait a minute, I thought Carr and Allen cannot be out together! If Allen is out, Brown must be out and Carr must be in.

- Notice that Speaker B might object in the same way to an utterance of (2) against background assumptions (a) and (b):

  (2) If Carr is out and Allen is out, Brown is in.

- Nothing we said so far accounts for this.

Stalnaker's constraint for indicative conditionals

- In any conversation, there is a *common ground* of propositions that are mutually accepted by the participants in the conversation.
- We may represent the common ground of a conversation as a set of possible worlds, the possible worlds in which all the propositions accepted by the participants in the conversations are true. Stalnaker calls this set of worlds *context set*.
- Stalnaker suggests that, in the case of indicative conditionals, the selection function is subject to the following constraint:

  IND. If w is a world of the context set and φ is the antecedent of an indicative conditional, $f(\varphi, w)$ must be a world of the context set.

- In other words, if it is applied to the antecedent of an indicative conditional and a world of the context set, the selection function must select a world in which the propositions accepted by the participants in the conversation are true.

The constraint at work

- The constraint explains why we accept (5) more readily than (6) (a fact observed by Adams 1970):

  (5) If Oswald did not shoot Kennedy, someone else did.

  (6) If Oswald had not shot Kennedy, someone else would have.

- Since it is common ground that Kennedy was shot, all the worlds in the context set are world in which someone shot Kennedy.
- Since (5) is an indicative conditional, the selection function, applied to the antecedent and a world of the context set, must select a world of the context set, namely a world in which Oswald did not shoot Kennedy, but someone did. Thus, (5) is true in any world of the context set. Thus, we should accept (5) in view of the assumptions we share.

Back to indicative Premise one

- Stalnaker's constraint on the selection function for indicative conditionals leads us to expect that there is something odd in asserting indicative Premise one against the background of (a) and (b):

  Premise one: if Carr is out, then if Allen is out Brown is in.

  (a) at least one barber must be in the shop
  (b) Allen always takes Brown with him when he goes out.

- If the participants in the conversation accept (a) and (b), the context set contains no worlds in which Car and Allen are out. Thus, (2) violates the constraint on the selection function for indicatives (and so does indicative Premise one if it has the same logical form as (2)):

  (2) If Carr is out and Allen is out, Brown is in.

- This accounts for why it is odd to assert (2) and Premise one, if we accept (a)-(b).

The execrably bad logician's narrative

- The execrably bad logician's narrative may now be summed up as follows:
  1. Natural language conditionals have Stalnaker's semantics.
  2. The logical form of (a) is (b):

     (a) if φ, then if χ, then ψ
     (b) if φ and χ, then ψ

  3. What's wrong in the proof of the indicative argument and in the proof of the subjunctive argument is step 2, since this step is no longer an instance of *modus ponens*.
  4. Premise one in the indicative argument is odd, because it violates the constraint on the selection function for indicative conditionals.

McGee's problem

- One welcome consequence of the bad logician's narrative is that it solves problem for Stalnaker raised by McGee (1985) in relation to sentences (7)-(8) and the following scenario:

  The scenario: A week before the 1980 U.S. presidential elections, the polls showed that Republican candidate Ronald Reagan was several points ahead of Jimmy Carter, the Democratic candidate. The other Republican in the race, John Anderson, was far behind in third position. In the end, Reagan won, Carter came second, Anderson a distant third.

  (7) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

  (8) If Reagan hadn't won the election, then if a Republican had won, it would have been Anderson.

- Given that there were only two Republicans in the race, (7) is true if uttered before the elections and so is (8) if uttered after the election. Yet, Stalnaker's abstract semantics incorrectly predicts that (7)-(8) are both false.

Solving McGee's problem

- Stalnaker's abstract semantics predicts that (7) is false, since the world *w* minimally different from the real world in which a Republican wins is a world in which Reagan wins and Carter comes second; thus the world minimally different from *w* in which Reagan doesn't win is a world in which Carter wins.

  (7) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

- Moreover, it also predicts that (8) is false, since the world *w* minimally different from the real world in which Reagan did not win is a world in which Carter won and Reagan came second: so, the world minimally different from *w* in which a Republican won is one in which Reagan won.

  (8) If Reagan hadn't won the election, then if a Republican had won, it would have been Anderson.

- However, if (7)-(8) have the same logical form as (9)-(10), Stalnaker's semantics correctly predicts them to be true: given that there are only two Republicans in the race, the closest world in which a Republican wins and it's not Reagan is a world in which Anderson wins.

  (9) If a Republican wins the election and it's not Reagan who wins it will be Anderson.

  (10) If Reagan hadn't won the election and a Republican had won, it would have been Anderson.

A narrative by a prominent semanticist

- A version of the solution to Carroll's paradox proposed by the bad logician is provided by Kratzer's (1981; 2012) semantics for conditionals.
- Let's see how.

Modal base and ordering source

- In Kratzer's semantics, truth is relative to two conversational backgrounds, f and g, where a conversational background is a function which assigns to each world a set of propositions.
- Background f is *the modal base*, which provides for each w the set of facts in w that are relevant for the interpretation of conditionals and modals.
- Background g is *the ordering source*, which for each w determines a similarity ranking of worlds with respect to w.
- Subjunctive conditionals select an *empty modal base* and a *totally realistic ordering source*, namely an ordering source that orders worlds according to how close they are to the world of evaluation.
- Indicative (epistemic) conditionals select a *realistic model base*, namely a modal base that assigns to each world a set of propositions which constitute a body of knowledge in that world.

Kratzer's semantics for conditionals

- Kratzer claims that bare conditionals have underlyingly form (i), where an adjoined *if*-clause modifies a sentence that has a necessity operator sitting in its left periphery:
  i. If ..., then NEC ...
- Kratzer's semantics is given in (K):

(K) For any conversational backgrounds f and g:

a. $[\![\mathit{if}\ \varphi\ \psi]\!]^{f,g} = [\![\psi]\!]^{f^*,g}$, where for all $w \in W$, $f^*(w) = f(w) \cup \{[\![\varphi]\!]^{f,g}\}$.

b. $w \in [\![\mathit{NEC}\ \alpha]\!]^{f,g}$ iff for all $u \in \cap f(w)$, there is a $v \in \cap f(w)$ such that (i) $v \leq_{g(w)} u$ and (ii) for all $z \in \cap f(w)$ if $z \leq_{g(w)} v$, then $z \in [\![\alpha]\!]^{f,g}$.

- Finally, Kratzer assumes that the logical form of stacked *if*-clauses like (a) is (b), where both antecedents φ and χ restrict the same modal operator:
  (a) if φ, then if χ, then ψ
  (b) (if φ, (if χ, (NEC ψ)))

Predictions

- By Kratzer's semantics, subjunctive Premise one is correctly predicted to be true in the circumstances described by Carroll relative to an empty modal base f and an ordering source g which orders worlds according to how close they are to those circumstances:

  Premise one: if Carr were out, then if Allen were out Brown would be in.

- Indeed, given the way f is updated, subjunctive Premise one is true relative to f, g in the circumstances described by Carroll iff "*NEC* Brown is in" is true relative to the modal base $f^* = \{$Carr is out, Allen is out $\}$ and the ordering source g. By the semantics of NEC in (Kb), "*NEC* Brown is in" is true relative to f^*, g in the circumstances described by Carroll iff (i) some world in which Carr is out, Allen is out and Brown is in is closest to these circumstances than any world in which Carr is out, Allen is out and Brown is in. Given that in the circumstances described by Carroll at least one barber is in, (i) is met and subjunctive Premise one is true in those circumstances.
- Indicative Premise one, on the other hand, has a natural epistemic interpretation in which (a)-(b) belong to the modal base f:

  Premise one: if Carr is out, then if Allen is out Brown is in.

  (a) at least one barber must be in the shop
  (b) Allen always takes Brown with him when he goes out.

  Under this interpretation, indicative Premise one presumably is correctly predicted to be infelicitous since f^*={(a), (b), Carr is out, Allen is out} is an inconsistent set.

Back to McGee's examples

- Notice that, for similar reasons, in Kratzer's system the correct interpretations are also achieved for McGee's examples:

(9) If a Republican wins the election and it's not Reagan who wins it will be Anderson.

(10) If Reagan hadn't won the election and a Republican had won, it would have been Anderson.

- Indeed, (9) is true relative to the modal base provided by McGee's scenario and an ordering source including facts concerning the normal course of events, since Anderson will win in all the worlds closest to $f(w_r) \cup$ {A Republican will win, Reagan will not win} (since Anderson will win in all the worlds closest to the worlds of the modal base in which a Republican will win and it will not be Reagan).
- Moreover, (10) is true relative to an empty modal base and a totally realistic ordering source, since Anderson won in all the worlds closest to the real world in which Reagan did not win and a Republican won.

The Kratzerian narrative

- The Kratzerian narrative about Carroll's paradox may now be stated as follows:
  1. Natural language conditionals have Kratzer's semantics.
  2. The logical form of (a) is (b), where both antecedents φ and χ restrict the same modal operator:
     (a) if φ, then if χ, then ψ
     (b) (if φ, (if χ, (NEC ψ)))
  3. What's wrong in the proof of the indicative argument, as in the proof of the subjunctive argument, is step 2, since conditionals contain no two-place conditional connective that could trigger the application of *modus ponens*: *if*-clauses act as adverbial modifiers that restrict operators (their role is to restrict the modal base of modal operator).
  4. Premise one in the indicative argument is odd, because the updating of the modal base by the conditional antecedents yields an inconsistent modal base.

The best account?

- The Kratzerian narrative, as well as the bad logician's narrative, captures the truth of the subjunctive premises, while explaining what is odd with the first premise of the indicative argument.
- Moreover, both narratives block the undesired conclusion that Carr cannot be out.
- Independent evidence for these narratives is also provided by the fact that they both solve the problem raised by McGee's examples (the link between McGee's problem and Carroll's paradox was observed by Helke 2018).
- So, they might seem to provide the best options to deal with Carroll's paradox.
- However, there are some serious problems with these narratives.

Explanatory inadequacy

- The bad logician's narrative crucially relies on the assumption that at a logical level nested conditionals like ⌜if φ, then if χ, then ψ⌝ are represented as ⌜if φ and χ, then ψ⌝.
- McGee (1985) objects to this move:

  > *The selective use of unnatural translations is a powerful technique for improving the fit between the logic of the natural language and the logic of a formal language. In fact, it is a little too powerful. One suspects that, if one is sly enough in giving translations, one can enable almost any logic to survive almost any counterexample. What is needed is a systematic account of how to give the translations. In the absence of such an account, the unnatural translations will seem like merely an ad hoc device for evading counterexamples.* (p. 471)

Descriptive inadequacy

- Both the bad logician's narrative and the Kratzerian narrative validate the equivalence of (a) and (b) (*the import-export law*):
  - (a) if φ, then if χ, then ψ
  - (b) if φ and χ, then ψ
- Yet, this equivalence does not hold unrestrictedly, as the following examples show.

Jack's height

- Suppose I want to test whether you can add or subtract centimeters. I do it by speculating on the height of Jack Aubrey. So, I ask you which of (11) and (12) is true:

(11) If Jack were 1 meter 80 tall, then if he were one cm taller, he'd be 1.81.

(12) If Jack were 1 meter 80 tall, then if he were one cm shorter, he'd be 1.81.

- Intuitively, (11) is true and (12) false. Clearly, (11) and (12) are not equivalent to (13) and (14):

(13) ??If Jack were 1 meter 80 tall and he were one cm taller, then he'd be 1.81.

(14) ??If Jack were 1 meter 80 tall and he were one cm shorter, then he'd be 1.81.

The Royal Navy

- I want to test whether Lea and Leo know the ranks in the Royal Navy. So, I say: suppose Jack is a captain, if he were a commodore instead, how higher would he be in rank? In response, Lea says (15) and Leo (16):

(15) If Jack is a captain, then if he were a commodore instead, he'd be one up in rank.

(16) If Jack is a captain, then if he were a commodore instead, he'd be one down in rank.

- Given the facts, (15) is true and (16) is false. Clearly, (15) and (16) are not equivalent to (17)-(18):

(17) ??If Jack is a captain and he were a commodore instead, he'd be one up in rank.

(18) ??If Jack is a captain and he were a commodore instead, he'd be one down in rank.

- (Examples (11)-(18) are from Zucchi 2011; **?**, 2021. Mandelkern 2021 reports a counterexample to import-export of the kind in (11)-(14) coming from S. Yablo).

Repairing the bad logician's narrative

- Suppose that conditionals whose surface form is (NC) may be projected at logical form as (i) or (ii) (where > is Stalnaker's conditional connective):

  NC. If A, then if B, then C
  - i. A B > C
  - ii. A>(B>C)

- In structure (i), unlike in (ii), the embedded "if" is not interpreted, it is simply a syntactic reflex of the higher "if", much like an embedded past tense in sequence of tense languages may be left uninterpreted, being simply a syntactic reflex of the highest past tense.
- Assuming that the antecedent in (i) is interpreted as a conjunction, options (i)-(ii) allow now for the possibility that, while subjunctive Premise one is interpreted as having conjoined antecedents, (11) is interpreted instead as a genuine case of nesting:

  Premise one: if Carr were out, then if Allen were out Brown would be in.

(11) If Jack were 1 meter 80 tall, then if he were one cm taller, he'd be 1.81.

The new problem

- The repair to the bad logician's narrative no longer stipulates that nested antecedents must project as conjunctions at LF, so it avoids McGee's *ad hocery* objection. Moreover, it avoids the descriptive inadequacy, since it no longer forces a conjoined reading of the antecedents in (11):

  (11) If Jack were 1 meter 80 tall, then if he were one cm taller, he'd be 1.81.

- But now, a new problem arises. The problem becomes that of explaining (a) why, given that Premise one appears to be true, it lacks the genuine nested reading (which as we saw makes the premise false), and (b) why, given that conjoining the antecedents should be possible for (11), (11) seems to lack such a reading.

  Premise one: if Carr were out, then if Allen were out Brown would be in.

- However, there are good reasons why speakers should select the conjoined reading of the antecedents for Premise one, but not for (11), in view of some reasonable principles governing suppositions. Let's see what they are.

No impossible suppositions

- I take it that P1 is a reasonable principle governing the rational practice of making suppositions:

  P1. Do not make suppositions such that it is impossible for them to be true.

- The principle is reasonable, since there is clearly something anomalous in supposing that something could obtain, which in fact could never obtain.
- (Sometimes we do make impossible suppositions, as when Gay-Lussac and von Humboldt wondered about the molecular structure of water or when we prove something by *reductio*. However, in these cases we assume that the supposition is an epistemic possibility or at least a pretended epistemic possibility).

Discarding possibilities

- Another reasonable principle is P2:

  P2. Do not discard a possibility and regard it as a live possibility right after.

- After all, if one opens up a possibility immediately after discarding it, what was the point of discarding the possibility in the first place?

Resolving conflicts

- It also seems reasonable to assume that violations of P1 (no impossible suppositions) are more serious than violations of P2 (discarding possibilities), since contradictory suppositions yield nonsense.
- This leads to the conclusion that clashes between P1 and P2, namely cases in which we have to choose between making impossible suppositions and discarding a possibility right after we introduced it, should be regulated by P3:

  P3. If P1 and P2 clash, violate P2.

Back to subjunctive Premise one

- Now consider subjunctive Premise one again:

  Premise one: if Carr were out, then if Allen were out Brown would be in.
- Suppose we select logical form (ii):
  ii. Allen is out>(Brown is out>Carr is in)
- In Carroll's scenario, this has the consequence that the second supposition (Brown is out) voids the immediately preceding one (Allen is out). It has this consequence, since the world closest to the circumstances Carroll described in which Brown is out is a world in which Allen is in.
- Thus, selecting structure (ii) for Premise one leads to a violation of principle P2, which requires that we do not discard a possibility and void it right after.
- On the other hand, logical form (i) does not violate P2 (or P1). So, structure (i) is selected:
  i. Allen is out Brown is out > Carr is in
- (McGee's examples may be dealt with in the same way).

Back to Jack's height

- Now consider (11) again:

(11) If Jack were 1 meter 80 tall, then if he were one cm taller, he'd be 1.81.

- Here, if we choose structure (ii), we would violate principle P1 (no impossible suppositions):
  ii. Jack is 1 meter 80 tall, Jack is one cm taller > Jack is 1 meter 81 tall
- Structure (i) violates principle P2 about discarding suppositions. Yet, by principle P3 about resolving conflicts between P1 and P2, this is preferable, so structure (i) is chosen:
  i. Jack is 1 meter 80 tall > (Jack is one cm taller > Jack is 1 meter 81 tall)
- (Notice: I am not claiming that violations of P2 originated by the need to abide by P1 and P3 always yield felicitous utterances. Whether they do or not depends on the communicative purposes of the conversation. In the context I described for (11), there is a point in making suppositions that cancel each other (what would Jack's height be if he were one cm taller that 1.81?) and the conditional becomes acceptable).

Import-export and indicatives

- As pointed out in Mandelkern (2021) and Zucchi (2021), the counterexamples to import-export involve the subjunctive mood, as the contrast between (15) and (19) shows:

(15) If Jack is a captain, then if he were a commodore instead, he'd be one up in rank.

(19) ??If Jack is a captain, then if he is a commodore instead, he is one up in rank.

- The obvious hunch to account for this contrast is to suppose that (19) is odd in virtue of the constraint on the selection function for indicatives.
- Since we expect that (19) should have structure (a), we need to make sure that the constraint applies to (a):
  a. Jack is a captain > (Jack is a commodore > Jack is one up in rank)

Derived contexts

- Following Stalnaker (2011), let's define the *derived context* for a conditional supposition φ made in a context c, for short $c(\varphi)$, as follows:

  $c(\varphi) = \{f(\varphi, w) : w \in c\}$.
- Let us now assume the following dynamic version of Stalnaker's semantics for conditionals:

  $[\![\textit{if } \varphi\textit{, then } \psi]\!]_{c,w} = 1$ iff $[\![\psi]\!]_{c(\varphi),w'} = 1$, where $w' = f(w, \varphi)$.
- We now correctly predict that (a) should violate the constraint on the selection function for indicatives.
  a. Jack is a captain > (Jack a commodore > Jack is one up in rank)
- The reason is that any world in c(Jack is a captain) is a world in which Jack is not a commodore, thus the selection function applied to the embedded antecedent "Jack is a commodore" and a world in c(Jack is a captain) cannot select a world in c(Jack is a captain).

The execrably bad logician's narrative revised

- The revised version I propose for the bad logician's narrative may now be summed up as follows:
  1. Natural language conditionals have the following dynamic version of Stalnaker's semantics:

     $[\![\textit{if } \varphi, \textit{then } \psi]\!]_{c,w} = 1$ iff $[\![\psi]\!]_{c(\varphi),w'} = 1$, where $w' = f(w,\varphi)$.
  2. In principle, a nested conditional (NC) may be projected at logical form as (i) or (ii) (which structure is chosen depends on pragmatic principles governing suppositions):

     NC. If A, then if B, then C
     i. A B > C
     ii. A>(B>C)
  3. Same as before.
  4. Same as before.
- Now, let's show how we can revise the Kratzerian narrative along similar lines.

The real work

- Notice that, in Kratzer semantics, the real work in accounting for the conjoined interpretation of antecedents in nested conditionals is done by semantic clause (a), which requires that the supposition introduced by the nested antecedent be added to the supposition introduced by the main antecedent:

(K) For any conversational backgrounds f and g:
a. $[\![\textit{if } \varphi\ \psi]\!]^{f,g} = [\![\psi]\!]^{f^*,g}$, where for all $w \in W$, $f^*(w) = f(w) \cup \{[\![\varphi]\!]^{f,g}\}$.
b. $w \in [\![\textit{NEC } \alpha]\!]^{f,g}$ iff for all $u \in \cap f(w)$, there is a $v \in \cap f(w)$ such that (i) $v \leq_{g(w)} u$ and (ii) for all $z \in \cap f(w)$ if $z \leq_{g(w)} v$, then $z \in [\![\alpha]\!]^{f,g}$.

- Indeed, Premise one would still be correctly predicted to be true even if we assumed structure (20):

Premise one: if Carr were out, then if Allen were out Brown would be in.

(20) if Carr were out NEC (if Allen were out (NEC Brown is in)).

Repairing Kratzer's semantics

- The problem is that Kratzer's semantics incorrectly predicts that (11) should only get a counterpossible interpretation:

(11) If Jack were 1 meter 80 tall, then if he were one cm taller, he'd be 1.81.

- What (11) suggests is that for subjunctive suppositions the updating of the modal base f is less constrained than it is for indicative suppositions, namely:
  - while for indicative antecedents $f^*(w) = f(w) \cup \{[\![\varphi]\!]^{f,g}\}$,
  - for subjunctive antecedents the option $f^*(w) = f(w)$ is also available.
- Assuming that principles P1-P3 govern the choice of how to update the modal base, this modification of Kratzer's semantics correctly predicts (21) to be true relative to an empty modal base f and an ordering source g which ranks worlds according to how close they are to the real world:

(21) if Jack were 1 meter 80 tall (if he were one cm taller (NEC he'd be 1.81)).

The Kratzerian narrative revised

1. Natural language conditionals have the semantics in (K'):

   (K') For any conversational backgrounds f and g:
   a. $[\![\textit{if } \varphi\ \psi]\!]^{f,g} = [\![\psi]\!]^{f^*,g}$, where for all $w \in W$, $f^*(w) = f(w) \cup \{[\![\varphi]\!]^{f,g}\}$ if φ is indicative; otherwise $f^*(w)$ may also be identical to $f(w)$.
   b. Same as in (K).
2. Same as before.
3. Same as before.
4. Same as before

Summing up

- I presented several different narratives about Lewis Carroll's barber shop paradox.
- In my view, among the narratives I considered, two fit the facts best: the revised bad logician's narrative and the revised Kratzerian narrative.
- The revised bad logician's narrative assumes that nested conditionals may project genuinely nested logical forms or logical forms in which the antecedents are conjoined.
- The revised Kratzerian narrative, on the other hand, achieves the same results by imposing conditions on the way the modal base is updated.
- Both narratives rely on a proposal about the pragmatic principles that govern our rational practice of making suppositions.

References I

Carroll, L. (1894). A logical paradox. *Mind*, 3(11):436–38.

Helke, T. (2018). On conditionals. *Philosophy: Faculty Publications*, Smith College, Northampton MA.

Jackson, F. (1979). On assertion and indicative conditionals. *The Philosophical Review*, 88(4):565–589.

Jackson, F. (1981). Conditionals and possibilia. *Proceedings of the Aristotelian Society*, 81:126–137.

Jackson, F. (1987). *Conditionals*. Blackwell, Oxford.

Johnson, W. E. (1894). A logical paradox. *Mind*, 3(12):583.

Khoo, J. (2013). A note on Gibbard's proof. *Philosophical Studies*, 166(1):153–164.

References II

Kratzer, A. (1981). The notional category of modality. In Eikmeyer, H. J. and Rieser, H., editors, *Words, worlds, and contexts. New approaches in word semantics.*, pages 38–74. de Gruyter, Berlin.

Kratzer, A. (2012). *Modals and conditionals*. Oxford University Press, Oxford.

Lewis, D. K. (1973). *Counterfactuals*. Harvard University Press, Cambridge, Massachusetts.

Lewis, D. K. (1976). Probabilities of conditionals and conditional probabilities. *The Philosophical Review*, 85(3):297–315.

Lewis, D. K. (1986). Postscript to probabilities of conditionals and conditional probabilities. In *Philosophical papers*, volume 2, pages 152–156. Oxford University Press, New York.

References III

Mandelkern, M. (2021). If p, then p! *The Journal of Philosophy*, 118(12):645–679.

McGee, V. (1985). A counterexample to modus ponens. *The Journal of Philosophy*, 82(9):462–471.

Russell, B. (1903). *The principles of mathematics*. Cambridge University Press.

Stalnaker, R. C. (1968). A theory of conditionals. In Rescher, N., editor, *Studies in Logical Theory*, pages 98–112. Blackwell, Oxford.

Stalnaker, R. C. (1975). Indicative conditionals. *Philosophia*, 5. Reprinted in Stalnaker (1999).

Stalnaker, R. C. (1999). *Context and content. Essays on intentionality in speech and thought*. Oxford University Press, Oxford.

References IV

Stalnaker, R. C. (2011). Conditional propositions and conditional assertions. In Egan, A. and Weatherson, B., editors, *Epistemic Modality*, pages 227–248. Oxford University Press, Oxford.

W. (1905). Lewis carroll's logical paradox. *Mind*, 14(2):292–93.

Williamson, T. (2020). *Suppose and tell. The semantics and heuristics of conditionals*. Oxford University Press, Oxford.

Zucchi, A. (2011). *Modus ponens* and minimal change conditionals. Abstract submitted to the Amsterdam colloquium.

Zucchi, A. (2021). Minimal change theories of conditionals and the import-export law. Paper presented at the University of Connecticut on May 7, 2021.