Exercises on indicative conditionals

Sandro Zucchi

2022-23

First exercise (the barbershop)

The problem

In "A logical paradox", published in 1894 in the journal *Mind*, Lewis Carroll tells the following story. In his village there are three barbers: Allen, Brown, e Carr. The three barbers run their shop according to these rules:

(a) at least one of the barbers must be in the shop at all times,

(b) Allen always takes Brown with him when he goes out.

One day, Uncle Jim and Uncle Joe are walking to the barbershop with their nephew. Uncle Jim hopes that Carr will shave him, since Brown is clumsy and Allen's hand has been shaky ever since he had the fever. Upon hearing Uncle Jim expressing this hope, Uncle Joe claims that Carr is certainly in the shop and provides the following argument in support of his claim:

Since one of the barbers must be in the shop, clearly

(1)　　if Carr is out, then if Allen is out Brown is not out.

Moreover, since Allen always takes Brown with him when he goes out, (2) is true:

(2)　　if Allen is out Brown is out.

Therefore,

(3)   Carr is not out.

The conclusion (3) follows from premises (1)-(2), since if we suppose that (1)-(2) are true and (3) is false, we arrive at a contradiction, namely we should conclude that both (2) and (4) are true:

(4)   if Allen is out Brown is not out.

Clearly, something must be wrong with Uncle Joe's argument, since facts (a) and (b) are compatible with the possibility that Carr is out and Allen and Brown are in, or that Carr and Brown are out and Allen is in. But what is wrong precisely? Carroll ends his paper wishing that "some of the readers of *Mind* who take an interest in logic will assist in clearing up these curious difficulties".

Your task

Help Lewis Carroll out:

1. Assume that indicative conditionals are material conditionals and represent (1)-(4) in the language LP.

2. Given the representation in LP, does the conclusion (3) follow from the premises (1)-(2)? Give a reason for your answer. (Hint: if you can't derive the conclusion from the premises, look for an assignment of truth values to the atomic formulae which makes the premises true and the conclusion false).

3. Given the representation in LP, is Uncle Joe right in claiming that (2) and (4) follow from (1), (2) and (5)? Give a reason for your answer.

   (5)   Carr is out.

4. Given the representation in LP, is Uncle Joe right in claiming that (2) and (4) contradict each other? Give a reason for your answer.

Second exercise (a proof of the existence of God)

Edgington (1986) mentions this argument (attributed to W. D. Hart):

If God does not exist, then it is not the case that if I pray my prayers will be answered (by Him). I do not pray. So God exists.

Answer the question: is the argument valid if indicative conditionals are material conditionals? Give a reason for your answer.

Third exercise (robustness)

Consider conditionals (6) and (7) again:

(6)   If New York is in New Zealand then 2 + 2 = 4.

(7)   If New York is in the United States then World War II ended in 1945.

Answer the question: are conditionals (6) and (7) robust with respect to their antecedent?

Fourth exercise (robustness)

Consider conditional (8) again:

(8)   If I am right, you are right or if you are right I am right.

As we saw, (8) is false if I assert the opposite of what you assert. This is a problem for the view that indicative conditionals are material conditionals, since if (8) were a material conditional it should be always true. Does the appeal to Jackson's robustness condition help here? Give a reason for your answer.