Possible Worlds Semantics for Belief Sentences

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1. Introduction

This paper is about possible worlds semantics for propositional attitude sentences. In particular I shall focus on belief reports in English such as “Lusina believes that tofu is nutritious.” It is well-known that possible worlds semantics for such reports suffers from the so-called problem of equivalence. In this paper I shall examine some attempts to deal with this problem and argue that they are unsatisfactory.

As I understand it, standard possible worlds semantics of belief reports contains these two theses. The intension thesis says that the semantic value of a that-clause is identical to the intension of its embedded sentence, and the binary thesis says that the verb “believe” expresses the binary relation believe between a subject and a proposition. Thus a sentence of the form “x believes that p” is true if and only if x stands in the believe relation to the proposition that is the intension of “p”. This leads to the well-known problem of equivalence. Consider these sentences:

(1) Sharifa believes that Mark Twain is Mark Twain.
(2) Sharifa believes that Mark Twain is Samuel Clemens.
(3) Sharifa believes that arithmetic is incomplete.
Since the embedded sentences of (1) to (3) are necessarily equivalent, they have the same intension, and so according to the two theses they have the same truth-conditions, which they don’t.  

2. Option 1: reject the binary thesis

Most responses to the problem of equivalence involve giving up the intension thesis. The other option of retaining the intension thesis and giving up the binary thesis is seldom pursued. Perhaps this is because on the face of it, “believe” certainly seems to express a binary relation. To give up the binary thesis one would have to adopt something like a hidden-indexical theory of belief (HIT). Roughly, according to such a theory, “believe” expresses a three-place relation between a subject, a proposition, and a contextually specified mode-of-presentation, under which the subject believes the proposition.

If HIT is correct, then the problem of equivalence can be solved. But whether HIT is correct is debatable, and even if it is, there are other issues to be sorted out. This is because HIT is compatible with a wide range of competing theories of propositions. You can accept HIT whether you take propositions to be sets of possible worlds, Russellian propositions, or structured intensions. So if HIT is correct, there is no reason to prefer the possible worlds approach, as far as the semantics of “believe” is concerned. In any case, I think those who try to repair the possible worlds approach do accept the binary thesis, and it is these attempts that I shall focus on.

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1 Here I assume a Kripkean semantics for proper names, so that “Mark Twain” and “Samuel Clemens” rigidly designate the same individual. I also assume, against authors such as Scott Soames and Nathan Salmon, that (1) and (2) in fact have different truth-conditions. But I shall not argue for this assumption here. Possible worlds semantics also faces the related problem of logical omniscience. This is a slightly different problem which is beyond the scope of this paper.

2 See for example discussion in Schiffer (1992) and Ludlow (1996).
3. The Reinterpretation Strategy

These attempts usually adopt one of these two strategies. The first I shall call the *reinterpretation strategy*, which can be found in Pietroski (1993) and Stalnaker (1987,88). The second, the *two-dimensional strategy*, develops out of two-dimensional modal logic. It is presented and defended in Chalmers (forthcoming).

The main idea of the reinterpretation strategy is that belief attribution to supposed to be heavily context-dependent, and in such a way that sometimes the semantic value of a that-clause is not the intension of its embedded sentence. So obviously it rejects the intension thesis. In Pietroski (1993) for example, the semantic value of a that-clause depends on whether the context in question is normal. He proposes that in a normal context, the semantic value of a that-clause is not the intension of its embedded sentence, but the conjunction of this intension and some additional meta-linguistic proposition. As for Stalnaker, he suggests that a process he calls “diagonalization” can sometimes identify correctly the semantic value of a that-clause, a proposition that is distinct from the intension of the embedded sentence.

One feature common to both approaches is that although they reject the intension thesis, they still take propositions to be individuated by their-truth-conditions. But I think this is a big mistake, for it means that if the semantic value of a that-clause is a proposition that is different from the intension of the embedded sentence, this proposition must have a modal status different from that of the intension. But it seems to me however context-sensitive belief attribution is supposed to be, it does not have such a radical consequence.

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3 Pietroski has now given up possible worlds semantics and become a Fregean. See Lau (1995) for additional criticisms of his earlier theory.
After all, the same that-clause can appear in both propositional-attitude and modal contexts, as in the following inference:

(4) Julian believes that arithmetic is complete.

That arithmetic is complete is necessarily false.

Julian believes something that is necessarily false.

It is hard to see how this can be a sound inference if Stalnaker or Pietroski is correct. If “that arithmetic is complete” denotes a contingent proposition, then presumably the second premise will be false. It cannot denote the necessarily false proposition either, because first of all on all possible worlds theory of belief nobody believes the necessarily false proposition, and if so the first premise cannot be true. Secondly, since there is only one necessarily false proposition in the possible worlds theory, to assign this proposition as the semantic value of the that-clause would mean that one is unable to distinguish the truth-conditions of this belief report from a sentence such as “Julian believes that there is a largest prime,” which also has a necessarily false embedded sentence. Finally, we also cannot claim that the two tokens of “that arithmetic is complete” have different semantic values, for the argument would presumably not be valid then. So it seems that on the reinterpretation strategy, (4) cannot be sound although there is no reason why it cannot be so.

4. The Two-dimensional Framework

I think the lesson here is that one cannot accept the binary thesis and reject the intension thesis, and still take propositions to be individuated by their truth-conditions. But if propositions are not individuated by their truth-conditions, does it not mean that possible worlds semantics has to be given up? This is where
the two-dimensional approach comes in. It accepts the idea that propositions can differ along some dimension other than their truth-conditions, such that there can be distinct but necessarily equivalent propositions. But the way these propositions are different from each other can still be explicated using possible worlds, and not by attributing to them intrinsic linguistic structure. David Chalmers has recently introduced just such a proposal, based on work done by Stalnaker and others. According to Chalmers, a belief or a thought is associated with two different intensions: a primary intension which is the narrow content of the belief or the thought, and a secondary intension which specifies its truth-conditions. If I understand Chalmers correctly, the problem of equivalence arises because we identify a proposition to be the more familiar secondary intension, when in fact it is a combination of both the secondary and the primary intensions.

It would be nice if such a theory does work out. The problem of equivalence will then be solved, with the bonus of having identified a theoretically important role for narrow content. It is beyond the scope of this paper to evaluate this dual-content theory of belief. What I shall do is to point to some serious difficulties in using this theory to solve the problem of equivalence.

To introduce Chalmers’ account, it would be useful to focus on concepts first. On the standard approach, the intension of a concept is given by what the concept refers to or picks out in each possible world. But according to Chalmers, this can be understood in two different ways, corresponding to the two different intensions of the concept. To specify the primary intension of a concept, we ask the following question: for each possible world \( w \), taking \( w \) to be the actual world, what does the concept refer to in \( w \)? Whereas to identify the secondary intension, we ask: given that the actual world is the one we are in, what is the referent of the concept for each possible world \( w \)?
Chalmers uses the concept of *water* as an example. The secondary intension is given by what the concept picks out at each world \( w \) given that the actual world is the one we are in. Since this rigid natural kind concept actually picks out \( \text{H}_2\text{O} \), it thereby picks out \( \text{H}_2\text{O} \) in every other world. Thus if we suppose that there are just two possible worlds, the actual world \( a \) and a counterfactual world \( c \), the secondary intension, as a function from worlds to referents, is something like \( \{ < a, \text{H}_2\text{O} >, < c, \text{H}_2\text{O} > \} \). But the primary intension of the concept is different though, given by what the concept of water picks out in each possible world \( w \) if it were the actual world. According to Chalmers, we fix the referent of the our concept of water by taking it to refer rigidly to whatever substance that is the dominant drinkable liquid in the actual environment. In the actual world it does pick out \( \text{H}_2\text{O} \), but if XYZ is such a liquid in possible world \( c \) then this is what the concept picks out (rigidly) in *if it were actual*. In such a case the primary intension would be the function \( \{ < a, \text{H}_2\text{O} >, < c, \text{XYZ} > \} \).

One might apply a similar procedure in identifying the primary and secondary intension of a sentence. Its primary intension is given by the truth-values the sentence has at each possible world \( w \) if \( w \) were actual. The secondary intension, which is simply “the intension” in the standard theory, is given by the truth-values the sentence has at each \( w \) given the actual world. Now suppose that \( a \) is the actual world, where the first celestial body appearing in the morning is the same as that which appears in the evening, namely Venus. But suppose that in possible world \( c \), the first celestial body that appears in the morning is Mars and that Venus is the first celestial body appearing the evening. If “Hersperus” rigidly picks out whichever celestial body appearing first in the morning, and “Phosphorus” rigidly picks out that which appears in he evening, then in such a case the primary intension of the necessary *a posteriori* “Hersperus = Phosphorus” might be \( \{ < a, \text{True} >, < c, \text{False} > \} \), with a secondary intension \( \{ < a, \text{True} >, < c, \text{True} > \} \). For the *a priori* necessary “Hersperus = Hersperus”, both the secondary and the primary intension would be \( \{ < a, \text{True} >, < c, \text{True} > \} \). Note that the primary intension of the first sentence is false at \( c \) because if \( c \) were actual, then “Hersperus” and “Phosphorus” picks out distinct objects. But the primary intension of the
second sentence is true at $c$ because whatever “Hersperus” picks out there it is identical with what it does picks out there, if that world were actual.

As can be seen from this example, it seems that we can distinguish the different cognitive status of two necessarily equivalent statements by their different primary intensions. Admittedly the presentation is rather sketchy, but I shall assume that the framework is coherent. The question that I shall now pursue is, whether the problem of equivalence can be solved by taking the semantic value of a that-clause to be some combination of both the primary and secondary intensions.

5. **The Appeal to Senses**

There are different ways to develop the present proposal. Unfortunately Chalmers did not provide any specific details, as his main aim was to defend his dualistic theory of belief-content, and the semantics of belief ascription was not his primary concern. But the simplest way of implementing his proposal would be to take the semantic value of a that-clause to be an ordered-pair consisting of both the primary and secondary intensions of its embedded sentence. There is no doubt the primary intension should be part of the semantic value under such a proposal, since it determines the truth-conditions and modal status of the that-clause. There is also no doubt that the secondary intension should play some role in individuating the semantic value, since the primary intension is not sufficient to distinguish necessarily equivalent propositions. I shall take this simple theory as the target of my criticism. I believe most of my objections will also be effective against other similar versions of the theory.

These objections I shall now present are not new in that they are familiar objections against a Fregean semantics of belief ascription. But I think they can also be directed against Chalmers account because the idea of primary intensions presupposes some notion of sense or mode of presentation. Recall that the
primary intension of a concept is supposed to specify what it takes for something at a possible world to be the referent of the concept if that world were actual. For example, the primary intension of the concept of water determines that the concept picks out the dominant, clear, drinkable liquid in our environment, and if XYZ is such a liquid in a counterfactual world \( w \), then this is what the concept picks out if that world were actual. Given the referent-determining role of the primary intension, I think it is appropriate to see it as some kind of sense.

Chalmers himself does acknowledge that his primary intension resembles a Fregean sense. He also clearly recognizes the familiar point that there is no such thing as the sense of an expression, and he suggests that different speakers might associate different primary intensions with an expression. But now here is the problem, if the primary intension of the embedded sentence of a that-clause can vary across speakers, then whose primary intension should we use as the constituent of the semantic value of the that-clause? Is it the one that the speaker associates with the sentence, or that of the subject to which belief is attributed? Or is it some other one to be determined by the context? Different answers will affect the identity of the semantic value in different ways. I shall now argue that none of them is satisfactory.

6. **Subject-based Approach**

To begin with, consider the proposal that the primary intension constituting the semantic value is one that the subject of the belief report associates with the embedded sentence. But one problem with this proposal is that as it stands it is incomplete. It is applicable only when two conditions are met: first, there is a specific individual to which belief is attributed, and second, this individual associates a certain primary intension with the embedded sentence of the belief report.
The problem is that belief attribution can take place even when one or both of these conditions do not obtain. Belief can be attributed in English to a non-English speaker who might not associate any primary intensions with the terms in the belief report, and in sentences such as (5) and (6), the that-clauses presumably do have some semantic value, even if no specific individual is referred to in the subject position:

(5) Nobody believes that Hersperus is Phosphorus.
(6) Everybody in Finland believe that snow is white.

In all these cases the present proposal fails to indicate how the semantic values of the that-clauses are determined.

But setting these problems aside, there is a more serious problem with allowing the semantic value of a that-clause to depend on the subject’s association. We should not forget that belief reports enter into inferences such as the following:

(7) Chris believes that the Hong Kong legislature is undemocratic.

Lavender believes everything that Chris Patten believes.

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Lavender believes that the Hong Kong legislature is undemocratic.

The validity of this inference can be readily explained if we take the argument to be of the following form:

(8) $B(c, p)$

$\forall x (B(c, x), \rightarrow B(1, x))$
This explanation of validity crucially depends on taking the two tokens of the same that-clause in (7) to have the same semantic value. But this might not be the case if different subjects can associate different primary intensions with the embedded sentence. On the present proposal then the inference would not be valid even though intuitively it is.\(^4\)

7. **Speaker-based Approach**

Here is a natural response to all the above objections: the primary intension which is part of the semantic value is the one that the speaker of the report associates with the embedded sentence, and not that of the subject to which belief is attributed. However, while this proposal does deal with the objections against the approach just discussed, it fails to solve the problem of equivalence. For this proposal does not rule out the possibility that the speaker associates the same primary intension with two distinct expressions, say two proper names “M” and “N”. In such a case his utterances “Daniel believes that M=M” and “Daniel believes that M=N” have the same truth-conditions, which they don’t.

One might of course insist that every speaker must associate distinct primary intensions with distinct expressions must have different senses. But it would be *ad hoc* to adopt such a principle without a more detailed justification. Furthermore, even if such a principle is true, the problem of equivalence will remain

\(^4\) A similar problem arises in accounting for the validity of this related inference: “Chris believes that the Chief Executive of Hong Kong is very conservative. John believes that the Chief Executive of Hong Kong is very conservative. Therefore there is something that both John and Chris believe.”
as long as there is the possibility of logical connection between distinct senses. As an example consider the following pair of sentences:

(9) Bing believes that diamonds are made of carbon.
(10) Bing believes that diamonds are diamonds.

Suppose that relative to the speaker of (9), the primary intension of “diamonds” picks out the hardest gemstones on earth, and that the primary intension of “carbon” picks out the stuff that makes up the hardest gemstones on earth. In such a case the primary intension of the embedded sentence of (9) would presumably be the same as that of (10). Furthermore, if we grant that “diamonds” refer rigidly to a form of carbon crystals, then the secondary intensions of the two embedded sentences will also be the same. If the semantic value of a that-clause is an ordered pair of primary and secondary intensions, then the semantic values of the that-clauses of (9) and (10) would have to be the same, even though they should not be.

8. **Context**

The upshot of the discussion is that neither a purely subject-based nor a purely speaker-based approach is correct. The obvious third alternative would of course be a context-based account. Perhaps the earlier objections can be avoided if the primary intension that in part makes up the semantic value of a that-clause depends in part on relevant contextual features.

However, I think the prospect of such an approach is not very good. In appealing to context-dependence, the only plausible contextual factor would be the speaker’s intention in uttering the belief report. But it seems to me that the speaker’s intentions would have to be more fine-grained and sophisticated than they actually are in order to deal with the problems I have raised. Take for example sentences (9) and (10),
presumably the speaker’s intention would have to be such that there is no necessary connection between
the primary intensions that are associated with “diamonds” and “carbon” respectively. We certainly are
conscious of no such intention in ordinary speech, and the burden of proof is upon the theorist who
nonetheless believe in the existence of such an intention. This is not to deny that a speaker might have
certain intentions as to what appropriate notions or concepts a subject should have if belief attribution is to
be successful. But it is doubtful whether such intentions can be captured by the apparatus of primary and
secondary intentions, and even if they can, since the present framework is parasitic upon some notion of
sense, one might wonder whether in such a case it is simpler to adopt some kind of Fregean account
instead. To substantiate these remarks would of course require more lengthy discussion, but I hope I have
shown that the two-dimensional approach has a long way to go before it can claim to have solved the
problem of equivalence.

References

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